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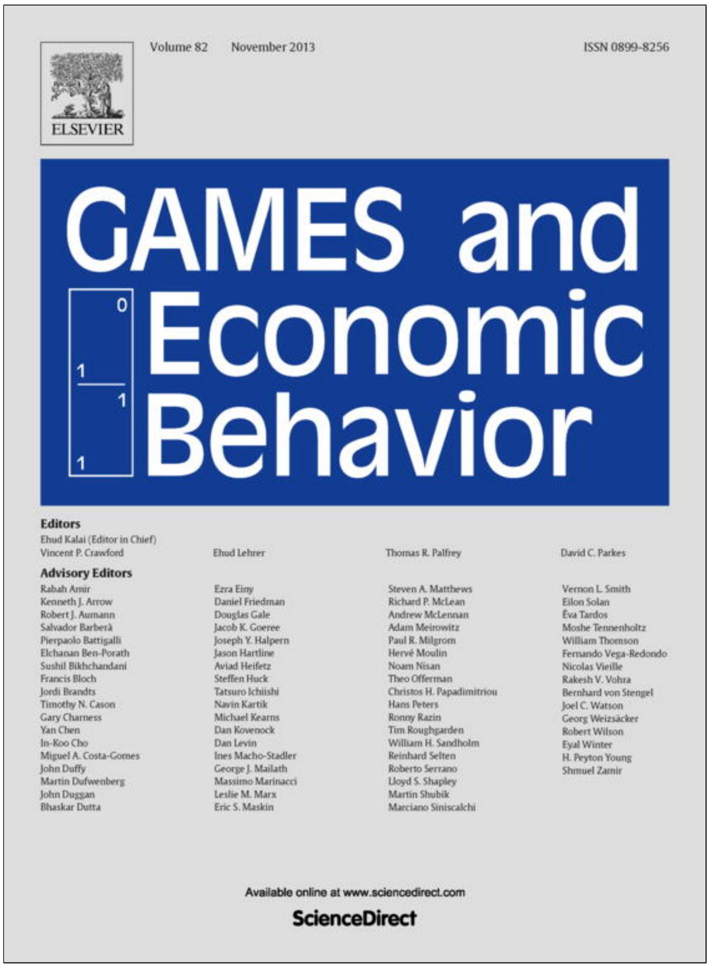

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Theoretical and experimental analysis of auctions with negative externalities ☆

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ABSTRACT

We investigate a private value auction in which a single "entrant" on winning imposes a negative externality on two "regular" bidders. In an English auction when all bidders are active, "regular" bidders free ride, exiting before price reaches their values. In a first-price sealed-bid auction incentives for free riding and aggressive bidding coexist, limiting free riding compared to the English auction. We find substantial, though incomplete, free riding in the clock auction. In first-price auctions, regular bidders bid more aggressively than the "entrant" and both bid higher than in auctions with no externality. Predictions regarding revenue, efficiency, and successful entry between the two auctions are satisfied.
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1. Introduction

The standard literature on auctions considers isolated markets with bidders that are *ex ante* identical and independent, so that losing bidders get a zero payoff (or the same payoff they have before the auction).[1] However, in cases where auctions take place within a broader economic framework this is not always the case, as auction participants may be competitors or cooperators in the relevant aftermarket. This paper considers the case where one of the competitors, on winning the auction, imposes a negative externality in the aftermarket. The negative externality is identity dependent, non-reciprocal, and on multiple competitors. We consider the simplest possible model to characterize all of these features: a single-object private value auction with three bidders where an "entrant," conditional on winning the item, imposes a negative externality on two (incumbent) "regular" bidders. An example is a takeover auction where one of the bidders is hostile, and the other bidders will be worse off if the hostile bidder wins. This negative externality is non-reciprocal since there is no externality if any of the non-hostile bidders win. Another example is a patent auction where all but one of the bidders are incumbents

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[1] See, for example, Milgrom and Weber (1982), Myerson (1981), Riley and Samuelson (1981) and Vickrey (1961).

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who already possess similar technologies, while the remaining bidder is a potential entrant. If the potential entrant wins, he will add more competition to the industry and take market share from the other bidders. On the other hand, if an incumbent wins, the market structure will remain more or less the same and no negative externality will be imposed on the other incumbents.

We examine the effect of a negative externality of this sort in both an English (clock) ascending price auction and a first-price sealed-bid (FPSB) auction. Intuitively, one might expect more aggressive (higher) bids in an auction with a negative externality. However, our equilibrium analysis shows that conditional on all three bidders being active in the clock auction, a regular bidder with a relatively low valuation will have incentive to drop out at a price lower than his value in an effort to free ride on a regular bidder with a higher valuation. However, once a regular bidder has dropped out, the remaining regular bidder will bid up to his value plus the absolute value of externality. In a sense, the clock auction provides a mechanism for the regular bidders to "coordinate" on when to free ride and when to bid aggressively. The FPSB auction, in contrast, provides no such opportunity because of no information revelation. In this case, both regular bidders bid more aggressively (higher) than the potential entrant, and the entrant in turn bids more aggressively than in an ordinary auction with no negative externality.

We conduct an experiment to examine whether the free-riding feature of the clock auction is present in the laboratory, as well as how closely subjects follow the other equilibrium predictions. In the clock auctions there is substantial, but far from complete, free riding on the part of regular bidders, which is roughly consistent with what the theory predicts. Further, in the clock auctions when two bidders are active, bids are close to equilibrium for regular bidders but not for entrants: Regular bidders drop out close to their value when the remaining bidder is also a regular, and at their value plus the externality when the remaining bidder is an entrant. While a number of entrants follow the dominant strategy of bidding up to their value, a considerable number consistently bid *above* their value. We relate this behavior to spitefulness, similar to results reported in Andreoni et al. (2007) in second-price auctions when bidders' valuations are common knowledge. In the FPSB auctions, consistent with theoretical predictions, regular bidders bid more aggressively (higher) than entrants and, as predicted, entrants tend to bid more aggressively compared to a FPSB auction without an externality. In the experiment, the clock auction generates higher efficiency and lower revenue than in the FPSB auction, consistent with the theory. Finally, entrants win more often in the FPSB auctions than in the clock auctions. Thus, to the extent one can draw policy implications from the present experiment, to encourage entry policy makers should adopt a FPSB auction rather than a clock auction.

There has been some theoretical work on closely related questions to the one investigated here. Jehiel and Moldovanu (1995) show that negative externalities may cause delays in negotiation, and Jehiel and Moldovanu (1996) investigate a case where a potential bidder cannot avoid the negative externality even if he does not participate in the auction. Jehiel et al. (1996) study mechanism design issues in auctions with negative externalities and show that the seller can sometimes obtain a greater profit by not selling the item.[2] Caillaud and Jehiel (1998) suggest that collusion will be imperfect if a buyer is worse off when his rival wins the object, to the point that the seller can design an auction to benefit from the (imperfect) collusive behavior of the bidders. Das Varma (2002) studies auctions with identity-dependent externalities which are one-to-one and are either reciprocal or non-reciprocal. Ettinger (2003) considers a situation where the losers of an auction care about the price paid by the winner as a result of various types of price externalities. He shows that a second-price auction can exacerbate the price externalities compared to a first-price auction. Finally, Hoppe et al. (2006) consider a license auction among both incumbents and entrants. They also demonstrate (albeit in a complete information setting) that free riding may arise due to potential competition among incumbents, which accounts for the counter-intuitive result that auctioning more licenses may not lead to a more competitive outcome.

To the best of our knowledge, we are the first to investigate free riding in an auction where one specific bidder can impose a negative externality on more than one bidder and to test the model experimentally. Regarding experimental work, Goeree et al. (2013) is closest in spirit to ours. They consider a situation where one bidder imposes a potential negative externality on two incumbent bidders in a multi-unit demand setting where neither incumbent can purchase the entire supply on her own. As such, regular bidders are faced with a threshold type problem. They focus on the incentive for demand reduction and preemptive bidding in both sealed-bid and ascending price auctions.

The rest of the paper is organized as follows: Section 2 establishes the theoretical framework. Section 3 describes our experimental design and procedures. Section 4 analyzes the data and presents the main results. Section 5 concludes.

2. Theoretical considerations

There is a single, indivisible object to be auctioned to three risk-neutral bidders. Each bidder's private value is assumed to be drawn independently and identically from a uniform distribution on $[0, 1]$. Two of the bidders are referred to as "regulars" or "incumbents" (R_1 and R_2) with private values v_1 and v_2. The third bidder is the potential entrant (E) with a private value v_E. There is an identity dependent negative externality of the amount $-x$ where $x \in (0, 1)$: if E wins the auction, both Rs receive a payoff of $-x$. However, if either R wins the auction, there is no externality so that losing bidders receive a zero payoff.

[2] Jehiel et al. (1999) analyze auctions with externalities following a multidimensional mechanism design approach.

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2.1. The English (clock) auction

In the English clock auction the price starts rising from zero. As the price rises, a bidder must decide whether to stay or drop out at the current price. The decision to drop out is irreversible. The auction ends when only one bidder is still active, who wins the item and pays the last drop-out price. We assume that the identities of bidders who have dropped out are common knowledge.

As it turns out, in our setting with negative externality, *both* Rs may want to drop out at price $P = 0$ if their values are sufficiently low. Unfortunately, employing the standard tie-breaking rule (ties broken at random) presents a technical challenge to equilibrium analysis.[3] As such we introduce an augmented auction, a second-price sealed-bid auction (SPSB) in which the high bidder wins the right to drop out, paying the second-highest price for the right to do so.[4] The losing bidder does not have to pay anything, but must continue in the auction for at least one price increment. The augmented auction is conducted after bidders know their valuations, and is only run if both bidders drop out at $P = 0$. Following the augmented tie-breaking auction, the two remaining bidders will compete for the rest of the auction (with probability one).[5]

Although the augmented auction is an unrealistic element for applications in field settings, we employ it since it is necessary to have a clear equilibrium benchmark against which to evaluate potentially interesting economic behavior. There are two potential problems with this solution. One is that it may have prompted subjects to free ride more often, as the procedure signals that dropping at $P = 0$ is something worth paying for.[6] This should be kept in mind when evaluating our results. Second, the artificial nature of the tie-breaking rule might be thought to impact the external validity of the results reported. Aside from the potential impact on the extent of the free riding, we do not believe this is a relevant consideration. In conducting the experiment we wanted to introduce a strong motivation for free riding. Our design does this. The fact that this introduces an artificial element that would not be replicated in any field setting is secondary to our goal of studying the impact of a strong motivation to free ride in auctions.[7] This is one of the strengths of the experimental method, being able to introduce in a controlled environment strong forces to see their impact on behavior.

Clearly, sincere bidding remains a weakly dominant strategy for the entrant; thus in equilibrium, the entrant drops out at the beginning of the auction with probability zero. Therefore, only the two Rs may form a tie at the zero price.

We will focus on symmetric increasing equilibria in which both incumbent bidders follow the same increasing bid functions in both the augmented tie-break auction and the English clock auction. In equilibrium, let $B(v_i)$ be incumbent i's drop-out price when the other two bidders are active and $\psi(v_i)$ be his bid in the augmented tie-breaking auction. We can show the following proposition:

Proposition 1. *There exists a unique symmetric increasing equilibrium in this English clock auction augmented by a tie-breaking auction at clock price $P = 0$. The equilibrium $\psi(\cdot)$ and $B(\cdot)$ are given below:*

For $x \in (0, 1/2)$,

$$\psi(v) = \frac{x^2 - v^2}{2}, \quad \text{for } v \in [0, x],$$

$$B(v) = \begin{cases} 0, & \text{for } v \in [0, x], \\ v - x, & \text{for } v \in (x, 1 - x], \\ 2v - 1, & \text{for } v \in (1 - x, 1]; \end{cases}$$

for $x \in [1/2, 1)$,

$$\psi(v) = \begin{cases} \frac{x^2 - v^2}{2}, & \text{for } v \in [0, 1 - x), \\ \frac{1}{2} - v, & \text{for } v \in [1 - x, \frac{1}{2}], \end{cases}$$

$$B(v) = \begin{cases} 0, & \text{when } v \in [0, \frac{1}{2}], \\ 2v - 1, & \text{when } v \in (\frac{1}{2}, 1]. \end{cases}$$

---

[3] In particular, there does not exist a symmetric equilibrium in which both Rs follow the same drop-out strategies: suppose R_2 drops when $v_2 \leqslant x$ in equilibrium. Then R_1 has an incentive to deviate to $B(v_1) > 0$ when v_1 is smaller than but sufficiently close to x.

[4] Alternatively, one could conduct an English clock auction between the two dropouts to determine who has the right to drop out at zero price. By introducing the augmented auction to break the tie, we effectively endogenize the tie-breaking rule to ensure the existence of equilibrium in the spirit of Jackson et al. (2002) and Simon and Zame (1990).

[5] In equilibrium there is zero probability that bidders drop out at any higher price. As such we can use an exogenously determined tie-breaking rule should this occur. The software was programmed to use a random tie-breaking rule.

[6] We are thankful to a referee for pointing this possibility out. We have no way of measuring the potential quantitative impact at this point. We do not believe it is large, but that is an empirical matter.

[7] More realistically, one can think of a scenario in which two long time incumbents collude to determine who drops out first, with the stronger of the two staying in as he has more resources with which to fight the entrant, splitting the cost of the fight after warding off entry. But this goes well beyond anything we have modeled or studied here.

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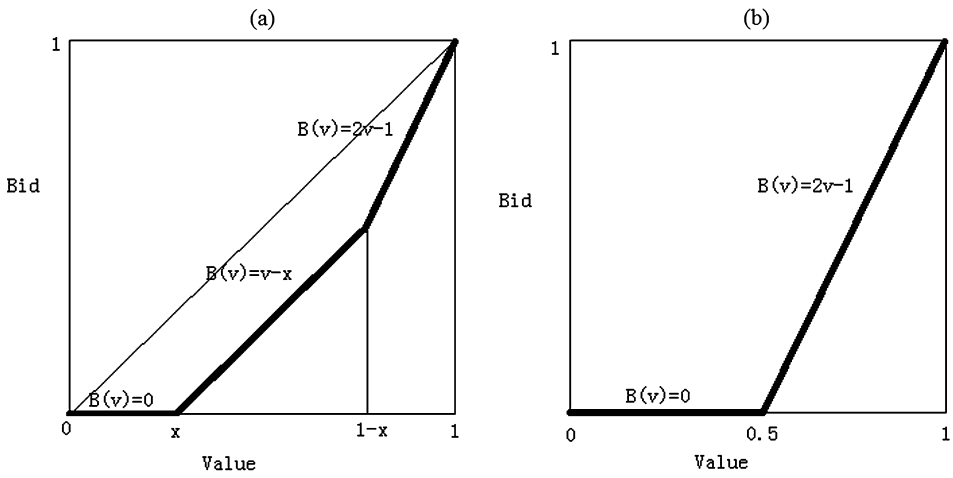

Fig. 1. Equilibrium first-drop price of R in clock auction: (a) $x < 0.5$; (b) $x \geqslant 0.5$.

When one bidder has already dropped out, R_i *with value* v *will stay until the clock price*

$$P = \begin{cases} v, & \textit{if the other remaining bidder is } R, \\ \min\{1, v + x\}, & \textit{if the other remaining bidder is } E. \end{cases}$$

The entrant stays till $P = v_E$.

Proof. See Appendix A. □

The equilibrium strategy of a regular bidder when all three bidders are active is shown in Fig. 1.

Clearly, regardless of the magnitude of the externality (x is small or large), $v > B(v)$ for $v \in (0, 1)$. Thus the equilibrium exhibits "free riding" in the sense that the lowest valued incumbent will drop out of the clock auction before the price reaches his or her value (and both may attempt to drop out at zero price). The complete proof of Proposition 1 is quite tedious, but the intuition is simple: instead of overbidding (and hence incurring a net loss) to prevent the entrant from winning, an incumbent would be better off by free riding on the other incumbent if the other incumbent has a better chance of beating the entrant. More precisely, this free-riding feature is caused by the combination of the negative externality and the dynamic nature of the clock auction: Without the dynamic nature of the clock auction, the incumbents simply cannot free ride, as will become clear after we develop the equilibrium for the FPSB auction.

Also note that $\psi(v)$ is strictly decreasing in v, so the endogenous tie-breaking rule (the augmented auction) is efficient in the sense that it will always select the incumbent with the higher value to stay, which improves overall efficiency in the auction.

2.2. The first-price sealed-bid auction

Again we will characterize the symmetric equilibrium $(\beta(\cdot), \gamma(\cdot))$ where $\beta(\cdot)$ is the equilibrium bid function for the two incumbents and $\gamma(\cdot)$ is the equilibrium bid function for the entrant.

Given that the other two bidders follow the proposed equilibrium strategies, incumbent 1 bids b to maximize his expected payoff:

$$\begin{aligned} E\Pi_1 &= F\big(\beta^{-1}(b)\big)F\big(\gamma^{-1}(b)\big)(v_1 - b) - x \int\limits_{\gamma^{-1}(b)}^{1} \int\limits_{0}^{\beta^{-1}(\gamma(v_E))} f(v_2) f(v_E)\, dv_2\, dv_E \\ &= \beta^{-1}(b)\gamma^{-1}(b)(v_1 - b) - x \int\limits_{\gamma^{-1}(b)}^{1} \int\limits_{0}^{\beta^{-1}(\gamma(v_E))} dv_2\, dv_E. \end{aligned}$$

That $\beta(\cdot)$ is a best response to $(\beta(\cdot), \gamma(\cdot))$ implies $\partial E\Pi_1/\partial b = 0$ when evaluated at $b = \beta(v_1)$. This leads to the following equation:

$$\beta'^{-1}\big(\beta(v_1)\big)\gamma^{-1}\big(\beta(v_1)\big)\big(v_1 - \beta(v_1)\big) + \gamma'^{-1}\big(\beta(v_1)\big)\big(v_1 - \beta(v_1)\big)v_1 - \gamma^{-1}\big(\beta(v_1)\big)v_1 + x\gamma'^{-1}\big(\beta(v_1)\big)v_1 = 0.$$

Similarly, the entrant bids r to maximize his expected profit:

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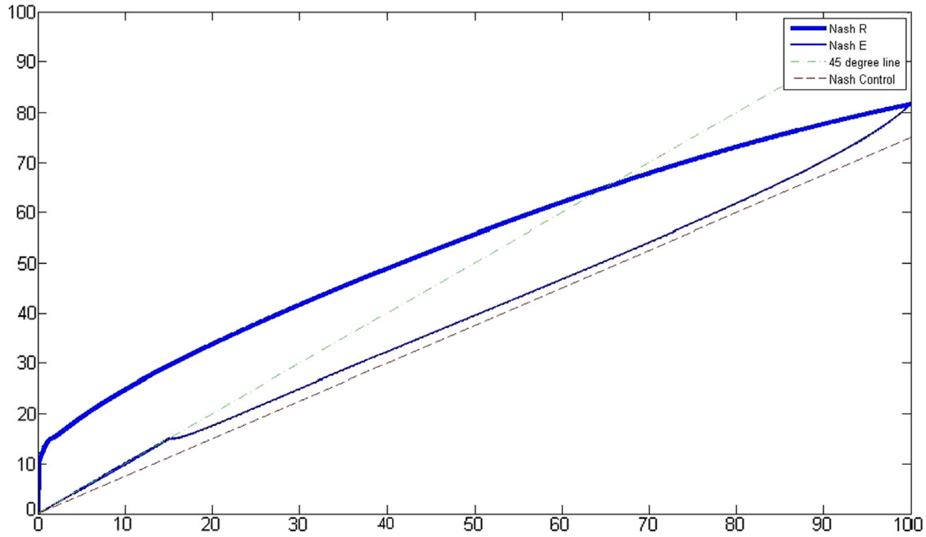

Fig. 2. Equilibrium schedules $\beta(\cdot)$, $\gamma(\cdot)$, and $b(\cdot)$ under FPSA ($x = 0.7$).

$$E\Pi_E = \left(F\left(\beta^{-1}(r)\right)\right)^2 (v_E - r) = \left(\beta^{-1}(r)\right)^2 (v_E - r).$$

That $\gamma(\cdot)$ is a best response to $\beta(\cdot)$ implies $\partial E\Pi_E/\partial r = 0$ when evaluated at $r = \gamma(v_E)$ or $\gamma^{-1}(r) = v_E$. This leads to the following equation:

$$2\beta'^{-1}\left(\gamma(v_E)\right) \cdot \left(v_E - \gamma(v_E)\right) - \beta^{-1}\left(\gamma(v_E)\right) = 0.$$

In equilibrium the following differential equations should hold simultaneously:

$$\begin{cases} \beta'^{-1}(b)\gamma^{-1}(b)(v_1 - b) + \gamma'^{-1}(b)(v_1 - b)v_1 - \gamma^{-1}(b)v_1 + x\gamma'^{-1}(b)v_1 = 0 \\ 2\beta'^{-1}(r)(v_E - r) - \beta^{-1}(r) = 0, \end{cases} \tag{1}$$

where $b = \beta(v_1)$ and $r = \gamma(v_E)$.

Proposition 2. *Under the first-price sealed-bid auction (FPSB), the symmetric equilibrium is characterized by the differential equations (1) and the boundary conditions $\beta(0) = \gamma(0) = 0$, and $\beta(1) = \gamma(1) = \bar{b}$ for some $\bar{b} \in (0, 1)$. For $v \in (0, 1)$, $\beta(v) > \gamma(v)$, i.e., incumbents bid more aggressively than the entrant in equilibrium.*

Proof. See Appendix A. □

Let the inverse bid functions be $\varphi_R(\cdot) = \beta^{-1}(\cdot)$ and $\varphi_E(\cdot) = \gamma^{-1}(\cdot)$. Eqs. (1) can be rewritten as follows.

$$\begin{cases} \varphi'_R(b)\varphi_E(b)\left(\varphi_R(b) - b\right) + \varphi_R(b)\varphi'_E(b)\left(\varphi_R(b) - b\right) - \varphi_R(b)\varphi_E(b) + x\varphi'_E(b)\varphi_R(b) = 0, \\ 2\varphi'_R(b)\left(\varphi_E(b) - b\right) - \varphi_R(b) = 0. \end{cases} \tag{2}$$

Fig. 2 plots the schedules $\beta(\cdot)$ and $\gamma(\cdot)$ (based on $x = 0.7$), along with the equilibrium bid function for a 3-bidder FPSB auction with no externality (given by $b(v) = \frac{2}{3}v$).[8] As shown $\beta(\cdot)$ lies above $\gamma(\cdot)$, as incumbents bid more aggressively than the entrant in order to avoid the externality. Moreover, the entrant's bid function lies above $b(v) = \frac{2}{3}v$, as the aggressive bidding of the incumbents heats up the competition, which in turn requires more aggressive bidding on the part of Es, more aggressive than under the risk-neutral Nash equilibrium absent an externality. From the figure, it is also clear that incumbents bid above their values when their values are below some threshold.

In what follows we will be comparing the FPSB and English auctions with respect to revenue, efficiency, and the probability that an entrant will win the auction.[9] Under the assumption of risk neutrality, revenue differences between the two

[8] Plotting $\beta(\cdot)$ forward starting at $v = 0$ is infeasible as $\beta'(0)$ cannot be determined. So we plot the (numerical) equilibrium bid schedules backward starting at $v = 1$. $\bar{b}$ is determined such that $\beta(0)$ and $\gamma(0)$ are sufficiently close to zero. That $x = 0.7$ is chosen as it is consistent with the parameter value used in our experiments.

[9] The results reported here are based on large sample simulations as there is no closed-form solution for the FPSB auction. These results will not necessarily hold for smaller sample sizes like those employed in the experimental sessions. As such, in comparing revenue, efficiency, and frequency that Es win the auction, we also report predicted outcomes based on the experimental valuations drawn. The most sensitive element with respect to small sample properties has to do with differences in average revenue. Differences in revenue variance never overlap for the sample sizes employed, with the English auction always more efficient than the FPSB auction as well.

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Table 1
Experimental treatments.

Session	Total number of subjects	Number of E subjects	Number of R subjects	Number of groups	Number of periods
Clock					
CL1	15	5	10	5	25
CL2	15	5	10	5	25
CL3	15	5	10	5	25
FPSB					
FP1	15	5	10	5	25
FP2	15	5	10	5	25
FP3	15	5	10	5	25
FPSB Ctrl					
FPC1	15	0	15	5	25
FPC1	15	0	15	5	25

auction formats increase monotonically with increases in the negative externality, with substantially higher variance in revenue in the English auctions throughout. With respect to efficiency as measured by the probability with which the bidder with the highest value wins the item (where value includes the externality for Rs), the English auction is always efficient. This follows from the fact that there will always be at least one R competing with the entrant, and this R will remain active up to his value plus the externality. Note, however, that this efficiency measure ignores the potential implications of entry for increased competition and increased efficiency in the product market after entry. Finally, the probability with which an E wins the auction is smaller than in the FPSB auction, as E's value must be above any R's value (including the negative externality) in order to win, but this is not the case in the FPSB auction.

3. Experimental design

Each experimental session consists of five auctions operating simultaneously with three bidders in each auction. There are three sessions each for the clock and FPSB auctions with externalities and two sessions for the FPSB with no externality (a control treatment). Instructions were read out loud with subjects having copies to follow.[10] Each session started with 3 dry runs followed by 25 paid periods. All subjects were paid their end of experiment cash balance. Table 1 shows the number of sessions along with the number of subjects under each auction format. Each session lasted for approximately one and a half hours.

Private values for all bidders were drawn *iid* from a uniform distribution with support $[0, 100]$ (with integer values only), with new values drawn before each auction. The externality was set at -70 throughout. At the beginning of a session subjects were randomly assigned to be either an E or an R (referred to as a type A and type B bidder, respectively), and remained in that role throughout. In each auction subjects were randomly assigned to a new three-bidder market, with each market containing one E and two Rs.

The clock auction employed a digital price clock starting at 0 and counting up by 2 every second. The computer screen showed a bidder's private value, the bidder's type, the current price of the item, and the type(s) of other active bidders. Drop-out prices and dropped bidders' types were reported as they occurred. Before the start of the auction each bidder had the opportunity to drop out at 0 or to bid in the auction. If more than one bidder dropped out at zero, an SPSB auction was conducted to decide the right to drop out at zero.[11] The auction stopped as soon as there was only one active bidder. This last bidder obtained the item and paid the price at which the next-to-last bidder dropped out. At the end of the auction, the price paid for the item and the winner's type were announced to all bidders, with earnings reported privately to each bidder. A complete history of these outcomes was available to each bidder as well.[12]

In the FPSB auction, each bidder entered an integer bid. The bidder with the highest bid obtained the item and paid a price equal to his bid. In the case of ties the computer randomly determined who got the item. Losing bidders each incurred a loss of 70 if E won, and zero profit if an R won. Subjects were permitted to bid above their valuations, with incumbents permitted to bid above their valuations plus the externality, although both of these outcomes were rarely observed.[13]

At the beginning of each session, Es were given an initial cash balance of 500 experimental currency units (ECUs) with Rs having a starting balance of 900 ECUs. The difference in initial cash balance was calibrated to account for losses due to the externality, and for expected differences in auction earnings between player types. These starting cash balances were private information so that Es would not have been aware of the larger starting cash balances for Rs. Cash balances were 500 ECUs in the FPSB control sessions. Subjects were paid their end of session balances in cash with ECUs converted into Chinese yuan at the rate of 10 ECUs = 1 yuan. Earnings averaged 72 (45) yuan for Rs and 52 (54) yuan for Es in the clock

10 A copy of the instructions along with screen shots can be found at http://www.econ.ohio-state.edu/kagel/Externality.

11 As noted, these procedures (or something similar to them) are needed to have a well-defined equilibrium.

12 The software was programmed using zTree (Fishbacker, 2007).

13 Entrants were also permitted to bid above their valuations plus the externality, as were subjects in the FPSB auctions who were permitted to bid up to 500 ECUs.

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(sealed-bid) auctions. Under the prevailing exchange rate this averages out to about $9 US dollars per subject.[14] Starting cash balances were sufficient to insure zero bankruptcies. All subjects had no previous experience with any type of auction experiment, although some of them may have had experience in another experiment.

An explicit control treatment was employed for the FPSB auction since subjects are known to bid well above the risk-neutral Nash equilibrium in the absence of a negative externality (see, for example, the many references cited in Kagel, 1995). As such a control treatment is needed to compare bidding with and without the externality. In contrast, bidding in English clock auctions absent externalities is known to converge to the dominant bidding strategy. This is confirmed here by bids in the clock auction when only two regular bidders remained active. The size of the externality employed was quite large as earlier experimental results under a similar design with a much smaller negative externality had a very limited impact on subject behavior, and provided little scope for learning.[15]

Subjects were recruited through posters from among the undergraduate students from various departments at Southwestern University of Finance and Economics in Chengdu, Sichuan Province, China. In 2011, Southwestern ranked 32 overall in China for undergraduate education, ranking 30th for freshmen quality based on Chinese college entrance exam scores.

4. Experimental results

4.1. Bidding in clock auctions

In the analysis that follows, unless stated otherwise, data will be reported for the last 12 auctions in each experimental session, after subjects have had some experience with the auction contingencies. Results are similar to those for the entire set of auctions, but somewhat closer to equilibrium outcomes, as there is some learning. Results for the entire set of auctions are reported in the online appendix to the paper.[16]

In what follows we report the experimental results in the form of a number of conclusions followed by the data supporting those conclusions.

Result 1. *In terms of first dropouts, there is substantial, but far from complete free riding on the part of Regular bidders* (*Rs*) *as the theory predicts.*

Figs. 3 and 4 show the first-drop price against values in the clock auctions for Rs and Es separately, along with the equilibrium bid functions.[17] There is a mass of bids at or close to zero, bids on the part of Rs with values in the interval $[0, 50]$ as the theory predicts: Rs with values less than or equal to 50 dropped before the clock auction started 34.7% of the time.[18] There are also a number of drops at, or close to value (the 45 degree line), representing a failure to free ride, even at low values. Although Rs' stage-one drops along the 45 degree line is not the free riding the theory predicts, it stands in marked contrast to the frequency with which Rs drop with bids *above* their value (or win the auction with bids *above* value) when competing with Es after stage one (see Figs. 6 and 7 below).

A closer examination of the data shows considerable heterogeneity in the extent to which different subjects drop at $P = 0$, as well as the fact that the probability of dropping at zero is inversely related to bidders' valuations. Over the last 12 auction rounds two out of thirty subjects (6.7%) always dropped out at zero with valuations less than or equal to 50.[19] Further, the cumulative percentage of Rs dropping at zero 50% of the time or more (including the two who always dropped at zero) is 33.3% (10/30). In contrast, 26.7% (8/30) of Rs never dropped at zero with these valuations. We ran a simple probit to determine the impact on the probability of dropping at zero with valuations less than or equal to 50 as a function of bidder valuations.[20] The coefficient value for valuations is negative and significant at the 1% level, with the implication that the probability of dropping at zero increases from 9.2% to 68% as valuations drop from 50 to 0. Although not what the theory predicts (everyone with these valuations should drop at 0), it's what one might expect from Rs as they have no possible way of solving precisely for the equilibrium outcome.[21]

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[14] This is a little higher than the average student wage which, for local college students with a standard work load averages between 10 and 20 yuan per hour. (The clock auctions averaged 2 h, with the sealed bid auctions lasting about 1.5 h.)

[15] See Hu et al. (2010) for these results. This experimental design used a random tie-breaking rule in case two or more bidders dropped out at the same time prior to the start of the auction. This does not result in a well-defined equilibrium bid function and was abandoned in favor of the present design. However, simultaneous drops prior to the start of these auctions were rare (5 out of 275 auctions) so the random tie-breaking rule had little impact on the outcomes. The size of the negative externality in this earlier experiment was 20, with values drawn from the support $[0, 100]$.

[16] http://www.econ.ohio-state.edu/kagel/Externality.

[17] This figure excludes the 10 cases in which a bidder dropped prior to the start of the auction and lost the SPSB auction.

[18] In contrast, when an R's value was greater than 50, he/she dropped out before the clock started less than 2% of the time. For Es with values less than or equal to 50, the overall frequency of dropping before the clock started was 13.9%. Both of these actions represent out-of-equilibrium play.

[19] These two had valuations less than or equal to 50 in two and four auctions respectively.

[20] The probit had a constant and bidders' valuations as the only explanatory variables. Standard errors were calculated with clustering at the individual subject level. All Rs were included in the probit any time they had a valuation of 50 or less.

[21] There were so few first drops by Rs according to the equilibrium prediction for valuations greater than 50 that we did not repeat this analysis for them. Note, sincere bidding on the part of these high valuation Rs is not unexpected given that bidders with valuations at the upper end of the interval $[0, 50]$ typically do not free ride (drop at zero as the theory predicts).

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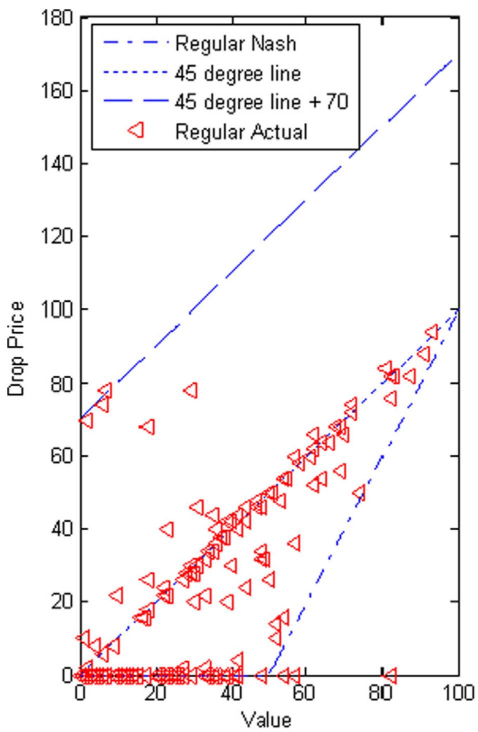

Fig. 3. First drop prices for Rs.

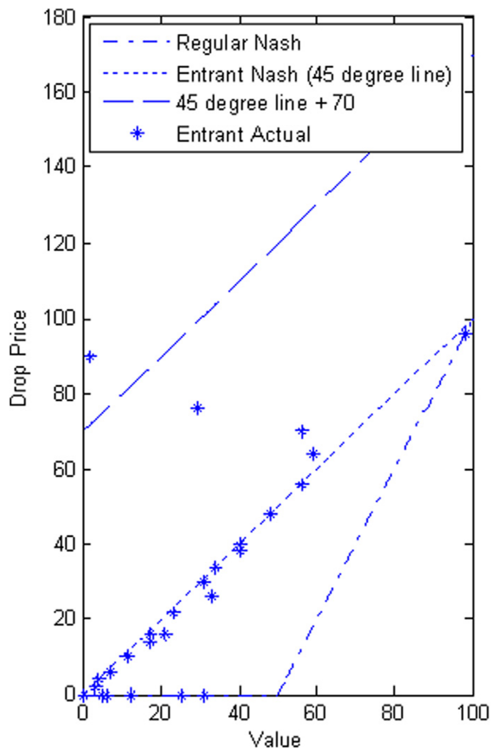

Fig. 4. First drop prices for Es.

Result 2. *The frequency with which both Rs drop out before the start of the auction is much less than predicted. The frequency with which the lower valued R wins the right to drop out in the tie-breaking auction is quite low as well, substantially lower than when neither R drops out, or only one R drops out, prior to the start of the auction. As a result efficiency is substantially greater in cases where both bidders fail to drop out prior to the start of the auction.*[22]

There were only 10 SPSB (tie-breaking) auctions in which both Rs had values less than or equal to 50 and both dropped out prior to the start of the auction, much less than the predicted number of simultaneous drops, 91.[23] In 3 of these 10 cases the SPSB auction achieved the efficient outcome, with the lower valued R winning the right to not participate in the auction.[24] In equilibrium in the SPSB auction bids are decreasing in value, so that a lower valued R should submit a higher bid in order to win the right to drop out. But Fig. 5 shows that bids in the SPSB do not decrease in value, although most of the SPSB bids are located below the equilibrium bid function curve. This failure to achieve consistently high efficiency in SPSB auctions is not surprising given the results from past Vickrey auctions (Kagel, 1995; Kagel and Levin, forthcoming). In contrast, when both Rs had values less than 50, but only one bidder dropped out prior to the start of the auction, the lower valued R dropped out first 71% of the time; and when neither bidder dropped out prior to the start of the auction, the lower valued R dropped first 62% of the time. While the latter is a direct consequence of the fact that many Rs who failed to drop at or near zero tended to bid up to their valuations, the former is not.

Result 3. *In clock auctions with two bidders being active, bids are close to equilibrium levels for Rs but not Es*: *Rs tend to drop out at their value when the remaining bidder is an R, and at their value plus the externality* (70) *when the remaining bidder is an E. While a number of Es followed the dominant strategy, a considerable number consistently bid above their values.*

Figs. 6 and 7 show, respectively, dropouts and winning bids for those sub-auctions where the remaining bidders were an E and an R. Two factors stand out. First, there are a large number of instances in which Es, contrary to the dominant bidding strategy, dropped out with bids above their values (68.7% of all Es dropping out second), but only a handful of auctions where Es wound up with a winning bid above their value (6.0% of these sub-auctions).[25] Second, there were large numbers of auctions in which Rs won with bids above their value (but less than the externality; 53.6% of these sub-auctions). There

[22] Given the low frequency with which both Rs dropped out prior to the start of the auction, the data reported on here is for all auctions.

[23] There were 7 cases where an R and E both dropped prior to the start of the auction, and one case in which all three bidders chose to drop prior to the start of the auction.

[24] In one case both Rs had the same value, thereby insuring an efficient drop out.

[25] Amending these calculations to allow for rounding error, or momentarily being distracted as the clock ticked up, to bidding above value + 4 ECUs, these percentages become 58.2% and 4.6%, respectively. Es won 17 auctions in total, with losses in 9 of the auctions. In 7 of these 9 auctions, Rs dropped out prior to bidding up to their value plus the externality. In equilibrium, Es would have won 1 of these 17 auctions.

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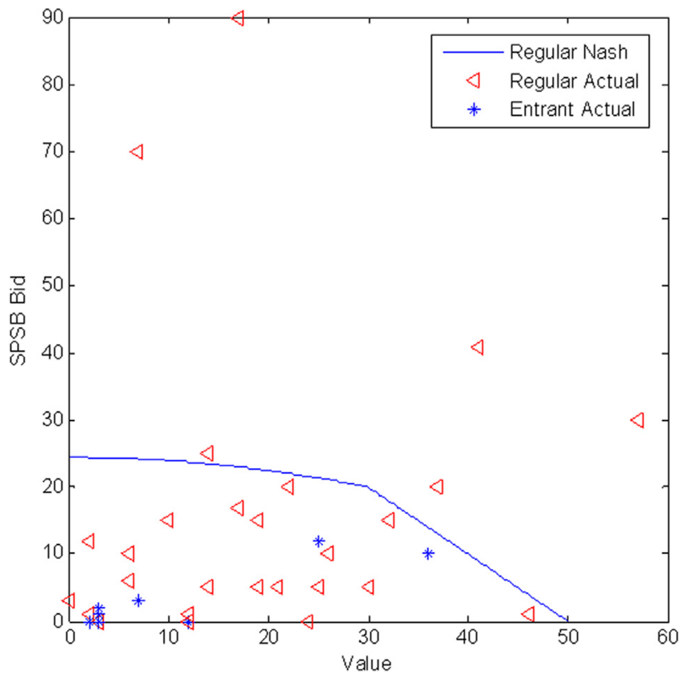

Fig. 5. Bids in SPSB auctions.

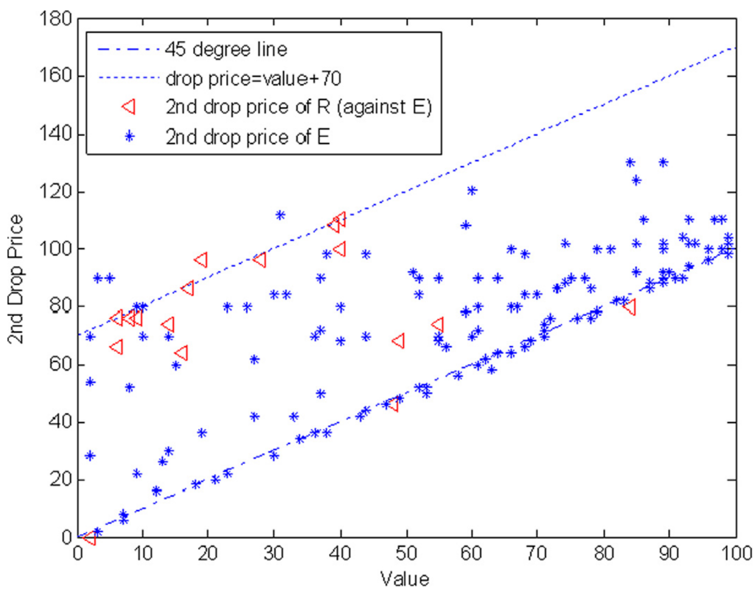

Fig. 6. Stage-two bids in clock auctions: both R and E active.

was some heterogeneity in the extent to which Es consistently bid in excess of their value, with 60.0% of Es bidding above their value more than 50% of the time.[26] In contrast, 100% of Rs either won or bid up to their value plus the externality more than 50% of the time.

Fig. 8 reports dropouts and winning bids for those sub-auctions where both bidders were Rs. In this case Rs' behavior is generally consistent with the dominant strategy as drop-out prices hover around the 45 degree line, and there were only two auctions in which Rs won with bids above their value when competing against another R.[27]

We were, quite frankly, surprised by the high frequency of Es bidding above their value. However, there is precedence for this in the literature: Andreoni et al. (2007) report a series of SPSB private value auctions under varying information about rivals' values. Most relevant to our experiment is their 1×4 auctions in which all four bidders had full information about each other's values, which they compared to their 4×1 treatment in which none of the bidders had any information

[26] This includes winning bids above value.

[27] Dropped from Fig. 8 are those sub-auctions in which when E dropped both Rs were still active with one or both bidding above their value. There is an obvious incentive in these cases for Rs bidding above value to drop immediately, which most of them did.

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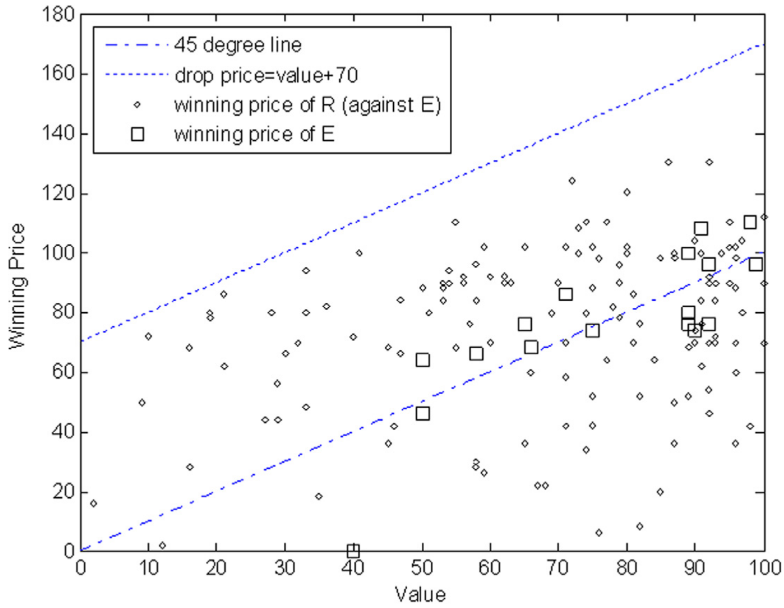

Fig. 7. Winning prices in clock auctions: both R and E active in stage 2.

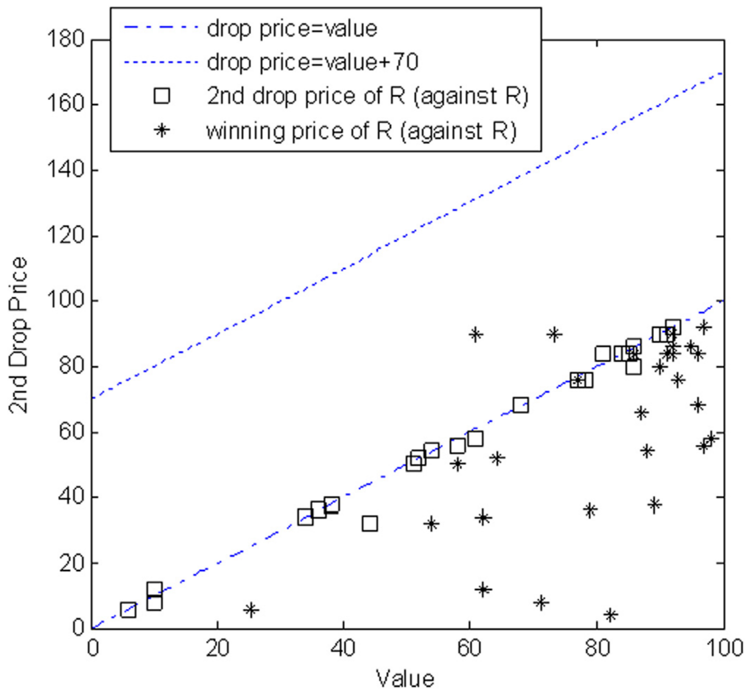

Fig. 8. Stage-two bids and winning prices in clock auctions: both Rs active.

about each other's values. Absent information about rivals' values 85.5% of all bids were sincere (equal to value) versus 62.5% sincere bidding in auctions with full information.[28] 12.0% were above value without information and 25.3% above value with full information. That is, with full information about rivals' valuations, there was a sharp increase in bidding above value which can be attributed to spiteful bidding. While Es in our auctions do not know Rs' values, they do know that in sub-auctions in which they are competing with an R, the R has an incentive to bid up to their value plus the amount of the externality. This allows Es to engage in spiteful bidding relatively safely as long as their bids stayed at or below 70, and to do so with added risk for bids above 70. Looking back at Fig. 6, this is consistent with this pattern, as Es bidding above value tapers off a bit for values above 70.[29] Finally, note that there are relatively few bids below value in Fig. 6, in

[28] Calculations are over the last 10 auctions out of the 20 conducted. Note, their subjects were undergraduates at the University of Wisconsin.

[29] When Es dropped second in these sub-auctions, their frequency of dropping above value plus 4 ECUs was 64.3% for values less than or equal to 70 and 48.0% for values above 70.

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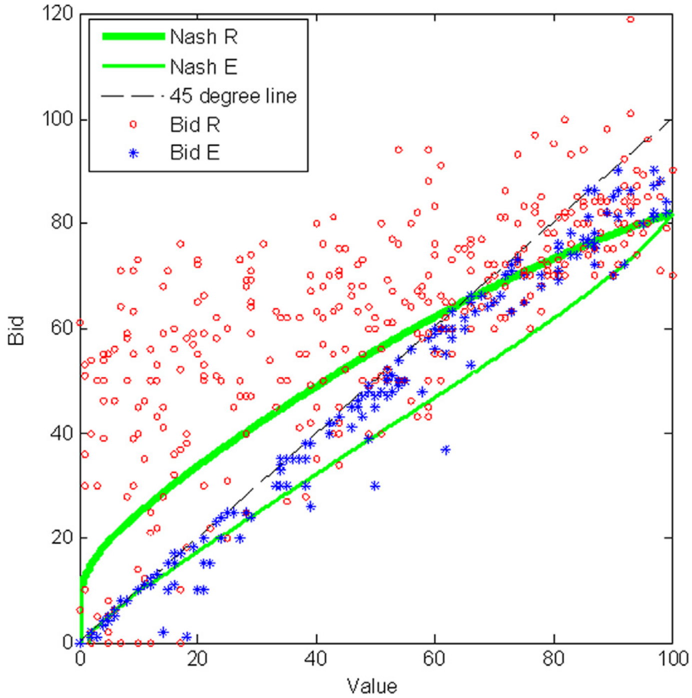

Fig. 9. Bids in FPSB auctions with externality present.

contrast to the 12.3% of bids below value reported in the Andreoni et al. full information treatment, which is suggestive of greater rivalistic bidding in China compared to Wisconsin.

4.2. Bidding in FPSB auctions

Result 4. *Consistent with the theory, Rs tend to bid more aggressively (higher) than Es in FPSB auctions. Also consistent with the theory, Rs and Es tend to bid more aggressively than in the FPSB independent private value auctions (the control treatment).*

Fig. 9 plots bids for Rs and Es in the FPSB auctions, along with the equilibrium bid functions. The graph shows that Rs bid higher than Es, on average, for all valuations, with Rs' bids at lower valuations closer to their value plus the externality than the risk-neutral Nash equilibrium (RNNE). Fig. 10 graphs bids for Rs compared to the controls, with Rs bidding higher than the controls, on average, at all valuations. Note that Fig. 10 shows the standard result for independent private value FPSB auctions—massive bidding above the RNNE, with Rs bidding even higher than that. Fig. 11 shows bids of Es compared to the controls. Es tend to bid higher than the controls, particularly at higher valuations. This occurs in spite of the rather massive overbidding relative to the RNNE in the controls. Finally, there is minimal bidding above value for Es and the controls, with no bids above their value plus the externality for Rs.[30]

Random effect regressions, with subject as the random component, reported in Table 2 confirm these results. In these regressions we have dropped bids for valuations less than 10 as (i) the equilibrium bid function with externalities has its most pronounced non-linear component in the interval $[0, 10]$, and (ii) at low valuations there is some tendency for "throw away" bids as subjects realize they have very little chance of winning the auction with very low valuations. Several specifications are reported, with and without a v^2 term. All the specifications treat the controls as the reference point against which to compare R's and E's bids. There is a separate dummy variable with value 1 if the subject is an R, and 0 otherwise, a separate dummy with value 1 if the subject is an E, and 0 otherwise, and interaction terms for each of the two dummies and v, and for the two dummies and the v^2 term. Although including the E^*v^2 and the R^*v^2 interaction effects shows that neither of these variables is statistically significant in their own right, and results in the E^*v interaction term no longer being statistically significant, a chi-square test shows that we can reject the null hypothesis at the 1% level that (i) the E^*v interaction terms and the E^*v^2 interaction terms are jointly equal to zero and (ii) the R^*v interaction terms and the R^*v^2 interaction terms are jointly equal to zero.

Fig. 12 plots the estimated bid functions for Rs, Es and the controls for the right-most specification in Table 2, our preferred specification. Evaluating the estimated bid function for this specification, Rs were bidding significantly more than the controls ($p < 0.05$) for all valuations, as the theory predicts. Similarly, Es were bidding significantly more than the controls ($p < 0.05$) for higher valuations ($v > 53$), with the differences between Es and the controls not significantly different from each other for values less than this. Finally, Rs were bidding significantly more than Es at lower valuations ($v < 78$), with no

[30] For Es 1.67% of bids were above value. For the controls, 0.56% of all bids were above value.

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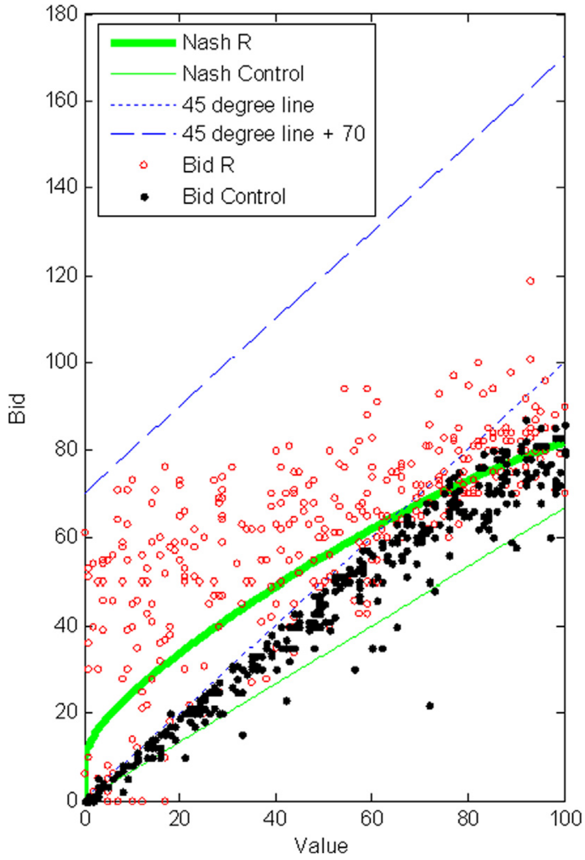

Fig. 10. Bids in FPSB auctions: Rs versus controls.

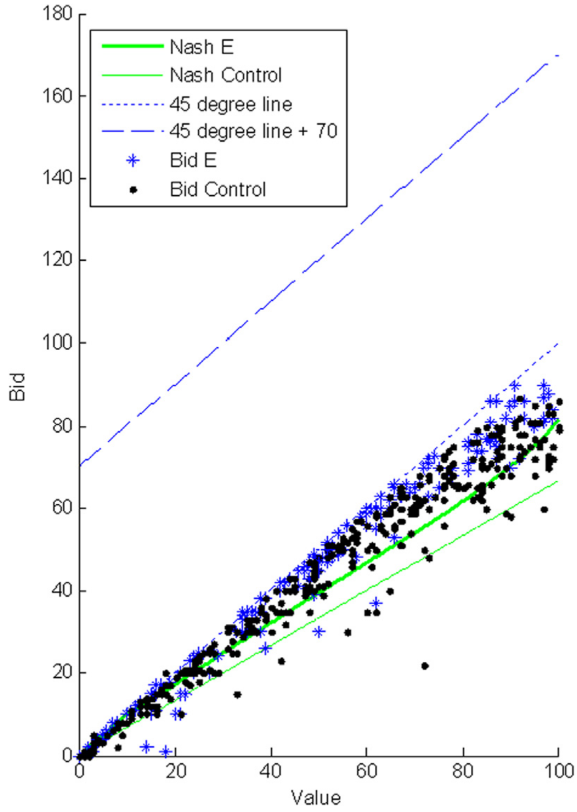

Fig. 11. Bids in FPSB auctions: Es versus controls.

Table 2
Random effect regressions. Dependent variable: Bids in FPSB auction.

Period	FPSB w/ & w/o externality		
	Value > 10		
	14–25	14–25	14–25
Constant	1.99*** (0.65)	−1.80 (1.17)	−3.25*** (1.13)
E dummy	−1.74 (1.55)	−1.89 (1.48)	−1.91 (2.63)
R dummy	33.95*** (3.62)	34.02*** (3.60)	37.73*** (4.70)
Value	0.81*** (0.02)	0.99*** (0.05)	1.06*** (0.05)
E × value	0.09*** (0.02)	0.09*** (0.02)	0.09 (0.11)
R × value	−0.29*** (0.05)	−0.30*** (0.05)	−0.47*** (0.14)
Value^2	–	−0.0016*** (0.0005)	−0.0022*** (0.0005)
E × value^2	–	–	0.000017 (0.000923)
R × value^2	–	–	0.0016 (0.0012)
Obs	802	802	802
R-sqrd	0.85	0.85	0.85

Standard deviations in parentheses.
*** Significant at 1 percent level, two tailed test.
** Significant at 5 percent level, two tailed test.
* Significant at 10 percent level, two tailed test.

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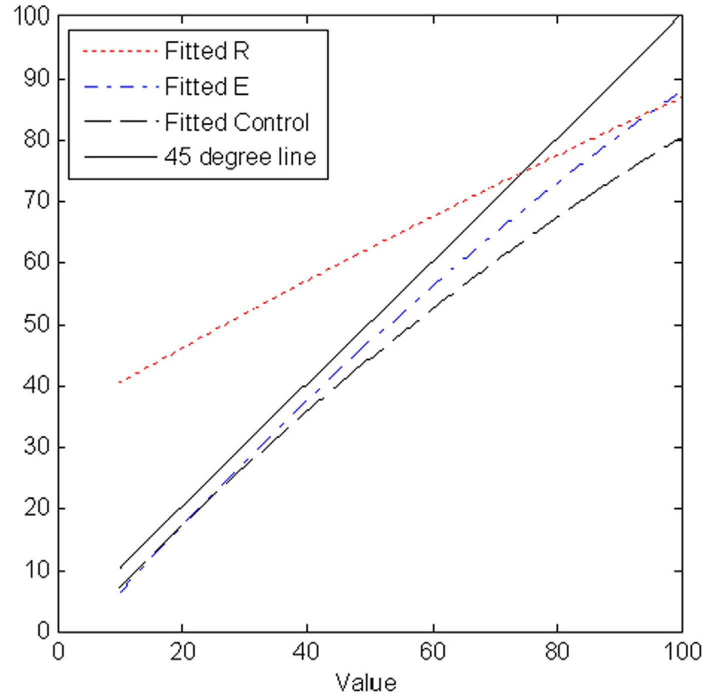

Fig. 12. Estimated bid functions for FPSB auctions: $v > 10$, including Vsq.

significant differences between the two at higher valuations. These results are all qualitatively consistent with the theory, since differences in bids between Es and the controls are minimal at lower valuations, with differences in bids between Rs and Es growing smaller at higher valuations. As a side note, the negative sign for the v^2 term reflects the fact that at the very highest valuations the tendency to bid well above the risk-neutral NE in IPV FPSB auctions tends to be moderated (see, for example, Dorsey and Razzolini, 2003).

4.3. Revenue and efficiency[31]

Result 5. *The FPSB auctions have higher average revenue and smaller variance in revenue than the clock auctions. The former is not statistically significant at conventional levels, but the latter is.*

Table 3 compares average revenue under the two auction formats where predicted revenue is based on auction valuations used in the experiment. Predicted revenue is higher under the FPSB auction than under the clock auction. Actual revenue is substantially higher than predicted revenue in the FPSB auctions, which is not unexpected given the overbidding (relative to the RNNE) typically found in FPSB auctions without externalities. Actual revenue is substantially higher than predicted revenue in the clock auctions as well. This is a result of Es bidding above value. Revenue is higher in the FPSB auctions than in the clock auctions, but this difference is not statistically significant at conventional levels, largely on account of bidding above value on the part of Es.[32]

Absent a negative externality, and assuming risk-neutral bidders, the variance in revenue in English auctions is predicted to be greater than in the FPSB auctions. With the negative externality this tendency is exaggerated as the remaining incumbent bidder is willing to bid up to his value to forestall entry, with the entrant bidding up to his value. This prediction is indeed satisfied in our experiment with the variance in revenue in the English auctions substantially higher than in the FPSB auctions (743.6 versus 130.7; $p < 0.01$).

Finally, as expected, average revenue is significantly higher in both the clock auctions and the FPSB auctions with the negative externality than in the FPSB no externality auctions ($p < 0.01$ in both cases).

Result 6. *The clock auctions are significantly more efficient than the FPSB auctions when the externality is present, and the FPSB control auctions are significantly more efficient than both auctions with the externality present.*

[31] Statistical tests throughout this section are based on OLS regressions in which the dependent variable consists of session average values for the variable in question and right hand side variables consist of dummy variables for the treatment conditions. For example, with revenue as the dependent variable, right hand side variables consist of a dummy variable for FPSB auctions with the negative externality = 1 (0 otherwise) and a dummy for the FPSB control auctions = 1 (0 otherwise), with the omitted treatment (English clock auctions) represented by the constant. Use of session value averages for the dependent variable represents the very conservative assumption that each auction session is a single observation because of complete autocorrelation of observations due to random re-mixing of subjects between auctions (see Frechette, 2012, for a discussion of statistical issues involved in, and alternative

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Table 3
Revenue, efficiency and percent of auctions E win.

	Ascending clock		FPSB		Difference	
	Actual	Predicted	Actual	Predicted	Actual	Predicted
Revenue	73.00 (2.03)	63.69 (2.12)	75.99 (0.87)	67.92 (0.84)	2.99	4.23
Efficiency	76.67 (3.16)	100.00 (0.00)	66.11 (3.54)	85.56 (2.63)	-10.56^{***}	-14.44
% E Win	9.44 (2.19)	0.56 (0.56)	20.00 (2.99)	17.78 (2.86)	10.56^{*}	17.22

Notes: Standard deviation in parentheses.
*** Significant at the 0.01 level.
* Significant at the 0.10 level.

We measure efficiency strictly in terms of the frequency with which the highest valued bidder wins the auction. In calculating this, Rs' values include the cost of the externality as well as their private value. In equilibrium the clock auction is predicted to be 100% efficient because free riding only exists in the first-stage of the auction, with bidders having a dominant strategy to bid up to their valuations after that. In contrast, the FPSB auction with the externality is akin to an auction with asymmetric valuations, so that efficiency will, in general, be less than 100%.

Table 3 reports average predicted and actual efficiency in the two auction formats with the externality present, where predicted efficiency is for the auction valuations actually drawn. Actual efficiency is significantly lower in the FPSB auctions than in the clock auctions, with the difference reasonably close to the predicted difference, in spite of the fact that absolute efficiency values are well below predicted levels in both cases. Note that the efficiency measure here excludes any potential increase in efficiency for the market in question given the predicted increase in entry for the FPSB versus the English auctions. A more complete measure of efficiency would take this effect into account.

Finally, the asymmetric nature of the FPSB auctions with the externality results in substantially lower efficiency compared to the FPSB control auctions (66.1% vs 88.3%, $p < 0.01$). The FPSB control auctions are significantly more efficient than the clock auctions as well ($p < 0.05$).

Result 7. *Es win more often in the FPSB auctions than in the clock auctions.*

Table 3 reports the proportion of auctions won by Es. Es are predicted to win substantially more often with the FPSB auctions compared to the clock auctions, with this result just failing to achieve statistical significance at the 5% level ($p = 0.052$). Given the weak power of this test due to the limited number of experimental sessions, it is worth noting that using session averages for all the auctions within a given experimental condition, entry is significantly greater in the English auctions at better than the 5% level. Thus, to the extent one can draw policy implications from the present experiment, our results indicate that if policy makers want to encourage entry they should adopt the FPSB auction rather than the clock auction.

5. Conclusion

This paper investigates theoretically and experimentally the effect of a negative externality on bidding strategies in an English clock auction and a first-price sealed-bid auction with two incumbents and one potential entrant. On the theoretical front, the equilibrium analysis shows that in the English auction one of the incumbents will typically engage in severe free riding. When this happens, the remaining incumbent bids quite aggressively to deter entry, bidding up to his value plus the potential cost of the negative externality. In the first-price sealed-bid auction, free riding and aggressive bidding coexist for incumbents as there is no way for bidders to implicitly coordinate their actions as in the English auction, resulting in incumbents bidding more aggressively than the potential entrant in order to avoid the externality. This in turn induces the entrant to bid more aggressively than in an auction with no externality present.

We observe substantial, though far from complete free riding in the English clock auction treatment. Further, in those sub-auctions where the remaining incumbent competes against the entrant, incumbents bid reasonably close to the equilibrium level predicted, well above their private value in order to deter the entrant. While bids are close to equilibrium levels for regulars they are not for entrants, with many of the latter bidding well above their valuations. Looking at the extant literature indicates that this is not some odd behavior of our sample population, but rather it reflects rivalistic bidding

ways of dealing with, the typical practice of re-mixing subjects between rounds in experiments). Given the clear theoretical predictions regarding efficiency and entry rates between the clock and FPSB auctions, one-tailed statistical tests are justified and used in Table 3.

[32] Es overbidding is present to begin with but grows substantially in frequency over time (36.4% of all E's bids in the first 13 auctions vs 58.3% in the last 12). As a result revenue is significantly higher in the FPSB auctions than in the clock auctions ($p < 0.01$) when calculated over all periods and over the first 13 periods.

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of the sort found in Andreoni et al. (2007) in second-price sealed-bid auctions when all bidders' valuations are common knowledge. Bidding in the first-price auctions, while well above the levels predicted under the risk-neutral Nash equilibrium, tends to satisfy the qualitative predictions of the equilibrium with regular bidders bidding higher than entrants, and both regular bidders and entrants bidding higher than in a first-price auction with no externality present. Qualitative predictions regarding higher revenue and lower efficiency in the sealed-bid versus clock auctions, along with the likelihood of the entrant winning the auction, are satisfied in the data as well.

Our model follows the literature using the term "externality" to measure the negative payoff to an incumbent when losing the auction. Perhaps a more proper term might be "post-auction effects." Extensions can be made to enrich the post-auction interactions, so that the "externality" would be a variable endogenously determined by the post-auction game. Investigation of this issue is left for future research.

Appendix A

A.1. Proof of Proposition 1

A.1.1. Derivation of $B(\cdot)$

It is obvious that remaining active until the price reaches his value is a (weakly) dominant strategy for the entrant (E). It is also clear that the equilibrium bidding strategies after one bidder has dropped out should be the ones specified in the proposition.

We will first derive the form of $B(\cdot)$ using the necessary equilibrium conditions (we will verify the sufficiency later). Suppose that incumbent 1 (R_1) drops at $B(\hat{v}_1)$ while incumbent 2 (R_2) follows $B(\cdot)$, where $\hat{v}_1$ is sufficiently close to v_1. Clearly, R_1 cannot benefit from dropping higher than $B(1)$ when the other two bidders are still active. So we only need to consider $\hat{v}_1 < 1$. Let $\Delta(v_1, \hat{v}_1)$ be the change in R_1's expected payoffs (from dropping at $B(\hat{v}_1)$ instead of $B(v_1)$). We first discuss the case where $x < 1/2$.

1. $v_1 \in [1-x, 1)$. If he deviates upwards, his payoff changes only when $v_2 \in (v_1, \hat{v}_1)$ and $v_E > B(v_2)$. If he deviates downwards, it only affects his payoff when he prevents another bidder (R_2 or E) from dropping between $B(v_1)$ and $B(\hat{v}_1)$. Hence,
$$\Delta(v_1,\hat{v}_1) = \begin{cases} \int_{v_1}^{\hat{v}_1}\int_{B(v_2)}^{1}(v_1 - v_E)\,dv_E\,dv_2, & \text{when } \hat{v}_1 > v_1, \\ \int_{\hat{v}_1}^{v_1}\int_{B(v_2)}^{1}(v_E - v_1)\,dv_E\,dv_2 + \int_{B(\hat{v}_1)}^{B(v_1)}\int_{B^{-1}(v_E)}^{v_1}(v_2 - v_1)\,dv_2\,dv_E, & \text{when } \hat{v}_1 < v_1. \end{cases}$$
For $B(\cdot)$ to constitute a symmetric equilibrium, we must have the following first-order conditions:
$$\lim_{\hat{v}_1 \to v_1^+}\frac{\partial\Delta(v_1,\hat{v}_1)}{\partial\hat{v}_1} = \frac{1}{2}\big[1 - B(v_1)\big]\cdot\big[2v_1 - 1 - B(v_1)\big] = 0,$$
$$\lim_{\hat{v}_1 \to v_1^-}\frac{\partial\Delta(v_1,\hat{v}_1)}{\partial\hat{v}_1} = \frac{1}{2}\big[1 - B(v_1)\big]\cdot\big[-2v_1 + 1 + B(v_1)\big] = 0.$$
Thus we must have $B(v_1) = 2v_1 - 1$ for $v_1 \in [1-x, 1)$.
2. $v_1 \in [r, 1-x)$, where r is the minimum value for an incumbent to drop above price zero. In other words, r is the (equilibrium) cutoff value under which an incumbent will drop out at the beginning of the clock auction (when the price equals 0). We do not impose any constraint on r for the moment.
If he deviates upwards and the dropped bidder happens to be R_2, we have $v_2 \in (v_1, \hat{v}_1)$, and the identities of the two remaining bidders will change from R_2 and E to R_1 and E and the deviation payoffs can be obtained accordingly; if the dropped bidder happens to be E, we have $v_E \in (B(v_1), B(\hat{v}_1))$, and the identities of the two remaining bidders will change from R_2 and E to R_2 and R_1. If she deviates downward, the situations can be analogously examined. Taking all together, we have
$$\Delta(v_1,\hat{v}_1) = \begin{cases} \int_{v_1}^{\hat{v}_1}\int_{v_1+x}^{v_2+x}(-x)\,dv_E\,dv_2 + \int_{v_1}^{\hat{v}_1}\int_{B(v_2)}^{v_1+x}(v_1 - v_E)\,dv_E\,dv_2, & \text{when } \hat{v}_1 > v_1, \\ \int_{\hat{v}_1}^{v_1}\int_{v_2+x}^{v_1+x}(-x - v_1 + v_E)\,dv_E\,dv_2 + \int_{\hat{v}_1}^{v_1}\int_{B(v_2)}^{v_2+x}(v_E - v_1)\,dv_E\,dv_2 & \\ \quad + \int_{B(\hat{v}_1)}^{B(v_1)}\int_{B^{-1}(v_E)}^{v_1}(v_2 - v_1)\,dv_2\,dv_E, & \text{when } \hat{v}_1 < v_1. \end{cases}$$
For $B(\cdot)$ to constitute an equilibrium, we must have
$$\lim_{\hat{v}_1 \to v_1^+}\frac{\partial\Delta(v_1,\hat{v}_1)}{\partial\hat{v}_1} = \frac{1}{2}\big\{\big[v_1 - B(v_1)\big]^2 - x^2\big\} = 0,$$
$$\lim_{\hat{v}_1 \to v_1^-}\frac{\partial\Delta(v_1,\hat{v}_1)}{\partial\hat{v}_1} = \frac{1}{2}\big[v_1 + x - B(v_1)\big]\cdot\big[-v_1 + x + B(v_1)\big] = 0.$$

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Since $B(v_1) = v_1 + x$ cannot be the equilibrium strategy, we must have $B(v_1) = v_1 - x$ for $v_1 \in [r, 1-x)$, which also implies that $r = x$.

For the case $x \geqslant 1/2$, the analysis is essentially the same except that there is a change in the supports of the piecewise function $B(v)$: the second segment now vanishes because $x \geqslant 1-x$. The lower bound of the second segment should now be $1/2$ instead of $1-x$ due to the continuity of $B(v)$, which implies that the incumbent with $v = 1/2$ should be indifferent.

Note that $B(v)$ so derived is unique. This means that should a symmetric equilibrium strategy exist, it must be uniquely determined for $v \geqslant x$. It's also worth noting that the uniqueness and the functional form of $B(\cdot)$ are independent of the tie-breaking rule at $P = 0$.

A.1.2. Derivation of $\psi(\cdot)$

Let $w(v)$ be the expected contingent payoff for an incumbent with value v who loses the augmented auction. We have

$$w(v) = \int_0^{\min\{1, v+x\}} (v - v_E)\, dv_E + \int_{\min\{1, v+x\}}^{1} (-x)\, dv_E = \begin{cases} \frac{(v+x)^2}{2} - x, & \text{for } v \leqslant 1-x, \\ v - \frac{1}{2}, & \text{for } v > 1-x. \end{cases}$$

Suppose $\psi(v)$ is strictly decreasing in v. R_1 with value v_1 bids $\psi(\hat{v}_1)$ to maximize

$$E\pi(\hat{v}_1, v_1) = \begin{cases} \int_0^{\hat{v}_1} w(v_1)\, dv_2/x + \int_{\hat{v}_1}^{x} [-\psi(v_2) + \int_{\min\{1, v_2+x\}}^{1} (-x)\, dv_E]\, dv_2/x, & \text{for } x < \frac{1}{2}, \\ 2\int_0^{\hat{v}_1} w(v_1)\, dv_2 + 2\int_{\hat{v}_1}^{\frac{1}{2}} [-\psi(v_2) + \int_{\min\{1, v_2+x\}}^{1} (-x)\, dv_E]\, dv_2, & \text{for } x \geqslant \frac{1}{2}. \end{cases} \tag{3}$$

The cutoff value for R_2 to drop out at zero price is x when $x < 1/2$ and $1/2$ when $x > 1/2$, and that is why we need to analyze the equilibrium separately for two cases above. This is illustrated by the equilibrium schedules in Section 2.1. In Eq. (3), $1/x$ is the density function of v_2 conditional on $v_2 < x$ (and 2 is the density function of v_2 conditional on $v_2 < 1/2$).

For $\psi(\cdot)$ to constitute a symmetric equilibrium, we must have $\partial E\pi(\hat{v}_1, v_1)/\partial \hat{v}_1|_{\hat{v}_1 = v_1} = 0$, which leads to, when $x < \frac{1}{2}$,

$$\psi(v) = \frac{x^2 - v^2}{2}, \quad \text{for } v \in [0, x],$$

and when $x \geqslant \frac{1}{2}$,

$$\psi(v) = \begin{cases} \frac{x^2 - v^2}{2}, & \text{for } v \in [0, 1-x), \\ \frac{1}{2} - v, & \text{for } v \in [1-x, \frac{1}{2}]. \end{cases}$$

For consistency check, $\psi(v)$ so derived is indeed decreasing in v. Also note that $\psi(v) = 0$ at $v = x$ when $x < 1/2$ and at $v = 1/2$ when $x \geqslant 1/2$.[33] Substituting the expressions of $\psi(v)$ into (3) and then differentiating $E\pi(\hat{v}_1, v_1)$ with respect to $\hat{v}_1$, we have

$$sgn\left\{\frac{\partial E\pi(\hat{v}_1, v_1)}{\partial \hat{v}_1}\right\} = \begin{cases} sgn\{(v_1 - \hat{v}_1)(\frac{v_1 + \hat{v}_1}{2} + x)\}, & \text{for } v < 1-x, \\ sgn\{v_1 - \hat{v}_1\}, & \text{for } v \geqslant 1-x, \end{cases}$$

which is positive when $\hat{v}_1 \leqslant v_1$ and negative when $\hat{v}_1 \geqslant v_1$. This shows that $\psi(v)$ given above is the unique symmetric equilibrium bid function in the augmented auction.

Therefore, if both incumbents are to drop at zero when their values are both below x when $x < 1/2$ and below $1/2$ when $x \geqslant 1/2$, then in the augmented auction no party has an incentive to deviate from bidding $\psi(\cdot)$ should the other follow $\psi(\cdot)$.

A.1.3. Verification of equilibrium $B(\cdot)$

We will consider incumbent 1 (R_1) whose value is v_1. Given that the other incumbent (R_2) follows $B(\cdot)$ and the entrant (E) stays till the price reaches his value, we will evaluate the change in his expected payoff by deviating to drop at $B(v_1 \pm \varepsilon)$ instead of $B(v_1)$, where $\varepsilon > 0$ and $v_1 \pm \varepsilon \in [0, \min\{v_1 + x, 1\}]$. Dropping at a price higher than $v_1 + x$ or 1 is obviously a dominated strategy. We will first examine the upward deviation, followed by the downward deviation.

We consider the case $x < 1/2$ first.

1. Upward deviation. R_1's deviation will affect the auction outcome only when it allows another bidder to drop first at $P \in (B(v_1), B(v_1 + \varepsilon))$. We discuss three sub-cases in order:

[33] It is easily seen that an incumbent with $v = x$ when $x < 1/2$ and $v = 1/2$ when $x \geqslant 1/2$ is indifferent between dropping out at zero and staying (but dropping out immediately after the clock starts). For incumbents who are not supposed to drop at price zero in equilibrium, their optimal bids in the augmented auction should be zero.

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1.1. $v_1 \in [0, x]$. R_1 is supposed to drop at $P = 0$. We will consider two possibilities for v_2: $v_2 \in [0, x]$ and $v_2 \in (x, 1]$ and show that it is not profitable for R_1 to deviate regardless of the value of v_2.

1.1.a. $v_2 \in [0, x]$. R_2 drops at zero in equilibrium. If $v_2 < v_1$, both the tie-breaking rule and the deviation to $B(v_1 + \varepsilon) > 0$ will make R_1 stay with E, and the auction outcome will stay the same. If $v_2 \geqslant v_1$, we already show that there is no incentive to deviate from $\psi(\cdot)$ conditional on dropping out, so we only need to rule out the possibility of dropping at a price strictly above zero. The auction outcome will be different if either of the two events occurs: the outcome changes from R_2 winning against E to E winning against R_1 or from R_2 winning against E to R_1 winning against E. Note that by this deviation, R_1 can avoid paying $\psi(v_2)$, which is his equilibrium payment in the augmented auction. The change in R_1's expected payoff is given by

$$\int_{v_1}^{x}\int_{v_1+x}^{v_2+x}(-x)\,dv_E\,dv_2+\int_{v_1}^{x}\int_{0}^{v_1+x}(v_1-v_E)\,dv_E\,dv_2+\int_{v_1}^{x}\beta(v_2)\,dv_2=-\frac{1}{3}(x-v_1)\big(2x^2-xv_1-v_1^2\big)\leqslant 0.$$

Therefore the deviation will make R_1 worse off if $v_2 \in [0, x]$.

1.1.b. $v_2 \in (x, 1]$. When the current price is 0, R_1's deviation to dropping at $B(v_1 + \varepsilon)(> 0)$ instead of 0 will affect the outcome only when it allows another bidder to drop first at $P \in (0, B(v_1 + \varepsilon))$. There are two possible cases: $v_1 + \varepsilon \in (x, 1 - x]$ and $v_1 + \varepsilon \in (1 - x, 1]$.

When $v_1 + \varepsilon \in (x, 1 - x]$, if R_2 drops first after the deviation, it implies that $v_2 \in (x, v_1 + \varepsilon)$ and $v_E \in (v_2 - x, 1]$. The two remaining bidders will change from R_2 and E to R_1 and E. The auction outcome could be affected in either of the two events: the outcome changes from R_2 winning against E to E winning against R_1; or from R_2 winning against E to R_1 winning against E. If instead E drops first after the deviation, we have that $v_E \in (0, v_1 + \varepsilon - x)$ and $v_2 \in (v_E + x, 1]$. The two remaining bidders will change from R_2 and E to R_2 and R_1. R_2 will win in both cases; thus R_1's payoff will be zero. To sum up, the change in R_1's expected payoff is given by

$$\int_{x}^{v_1+\varepsilon}\int_{v_1+x}^{v_2+x}(-x)\,dv_E\,dv_2+\int_{x}^{v_1+\varepsilon}\int_{v_2-x}^{v_1+x}(v_1-v_E)\,dv_E\,dv_2=\frac{1}{2}\int_{x}^{v_1+\varepsilon}(v_1-v_2)\cdot(v_1-v_2+4x)\,dv_2<0.$$

Since $v_2 \in [v_1, v_1 + \varepsilon]$ and $\varepsilon \leqslant x$, we have $v_1 \leqslant v_2$ and $v_1 - v_2 \geqslant -x$. Therefore R_1 will be worse off from the deviation. When $v_1 + \varepsilon \in (1 - x, 1]$, the argument is similar as above except that $2v_2 - 1$ is R_2's bid function when $v_2 \in (1 - x, v_1 + \varepsilon)$. It can be demonstrated analogously that the change in R_1's expected payoff is also negative.

1.2. $v_1 \in (x, 1 - x)$. We will consider two possible cases: $v_1 + \varepsilon \in (v_1, 1 - x)$ and $v_1 + \varepsilon \in [1 - x, 1)$. ($v_1 + \varepsilon = 1$ is not possible because of the restrictions that $v_1 < 1 - x$ and $\varepsilon \leqslant x$.)

1.2.a. $v_1 + \varepsilon \in (v_1, 1 - x)$. If R_2 drops first after the deviation, it implies that $v_2 \in (v_1, v_1 + \varepsilon)$ and $v_E \in (v_2 - x, 1]$. The identities of the two remaining bidders will change from R_2 and E to R_1 and E. Such a deviation can change the outcome in two possible events: the outcome changes from R_2 winning against E to R_1 winning against E, and R_1's payoff changes from 0 to $v_1 - v_E$; from R_2 winning against E to E winning against R_1, and R_1's payoff changes from 0 to $-x$. If E drops first after the deviation, it implies that $v_E \in (0, v_1 + \varepsilon - x)$ and $v_2 \in (v_E + x, 1]$, and the identities of the two remaining bidders will change from E and R_2 to R_1 and R_2. The fact that R_2 is active when the price has reached $B(v_1)$ implies that $v_2 \geqslant v_1$. R_2 will win in both cases and R_1's payoff is not affected by the deviation. Therefore, the change in R_1's expected payoff is given by

$$\int_{v_1}^{v_1+\varepsilon}\int_{v_2-x}^{v_1+x}(v_1-v_E)\,dv_E\,dv_2+\int_{v_1}^{v_1+\varepsilon}\int_{v_1+x}^{v_2+x}(-x)\,dv_E\,dv_2=\frac{1}{2}\int_{v_1}^{v_1+\varepsilon}(v_1-v_2)\cdot(v_1-v_2+4x)\,dv_2,$$

which is negative for the same reason as above. Therefore R_1 will be worse off from the deviation.

1.2.b. $v_1 + \varepsilon \in [1 - x, 1)$. If R_2 drops first after the deviation, it implies that either $v_2 \in (v_1, 1 - x)$ and $v_E \in (v_2 - x, 1]$, or $v_2 \in [1 - x, v_1 + \varepsilon)$ and $v_E \in (2v_2 - 1, 1]$. The identities of the two remaining bidders will change from R_2 and E to R_1 and E. Such a deviation can change the outcome in two possible ways: the outcome changes from R_2 winning against E to R_1 winning against E, and R_1's payoff changes from 0 to $v_1 - v_E$; from R_2 winning against E to E winning against R_1, and R_1's payoff changes from 0 to $-x$. If bidder E drops first after the deviation, it implies that either $v_E \in (0, 1 - 2x)$ and $v_2 \in (v_E + x, 1]$, or $v_E \in [1 - 2x, 2(v_1 + \varepsilon) - 1)$ and $v_2 \in (\frac{v_E+1}{2}, 1]$. The identities of the two remaining bidders will change from E and R_2 to R_1 and R_2. The fact that R_2 is active when the price has reached $B(v_1)$ implies that $v_2 \geqslant v_1$. R_2 will win in both cases and R_1's payoffs will both be zero. Combining these two cases, the change in R_1's expected payoff is given by

$$\int_{v_1}^{1-x}\int_{v_2-x}^{v_1+x}(v_1-v_E)\,dv_E\,dv_2+\int_{v_1}^{1-x}\int_{v_1+x}^{v_2+x}(-x)\,dv_E\,dv_2+\int_{1-x}^{v_1+\varepsilon}\int_{2v_2-1}^{v_1+x}(v_1-v_E)\,dv_E\,dv_2+\int_{1-x}^{v_1+\varepsilon}\int_{v_1+x}^{1}(-x)\,dv_E\,dv_2$$

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$$= \frac{1}{2}\int_{v_1}^{1-x}(v_1-v_2)\cdot(v_1-v_2+4x)\,dv_2 + x\int_{1-x}^{v_1+\varepsilon}(v_1+x-1)\,dv_2$$
$$+\frac{1}{2}\int_{1-x}^{v_1+\varepsilon}\left[v_1^2-x^2+(2v_1-2v_2+1)\cdot(1-2v_2)\right]dv_2,$$

which is negative for the same reason as above. Therefore R_1 will be worse off from the deviation.

1.3. $v_1 \in [1-x, 1)$. If R_2 drops first after the deviation, it implies that $v_2 \in (v_1, v_1+\varepsilon)$ and $v_E \in (2v_2-1, 1]$. The identities of the two remaining bidders will change from R_2 and E to R_1 and E. Such a deviation will change the auction outcome from R_2 winning against E to R_1 winning against E, and R_1's payoff changes from 0 to $v_1 - v_E$. If E drops first after the deviation, it implies that $v_E \in (2v_1-1, 2(v_1+\varepsilon)-1)$ and $v_2 \in (\frac{v_E+1}{2}, 1]$. The identities of the two remaining bidders will change from E and R_2 to R_1 and R_2. R_2 will win in both cases and R_1's payoffs will both be zero. The change in R_1's expected payoff is thus given by

$$\int_{v_1}^{v_1+\varepsilon}\int_{2v_2-1}^{1}(v_1-v_E)\,dv_E\,dv_2 = 2\int_{v_1}^{v_1+\varepsilon}(1-v_2)(v_1-v_2)\,dv_2,$$

which is non-positive because $v_1 \leqslant v_2$ and $v_2 \leqslant 1$. Therefore when $v_1 \in [1-x, 1)$, R_1 will not be better off from an upward deviation.

We have thus shown that for all $v_1 \in [0, 1]$, incumbent 1 will not be better off from any upward deviation.

2. Downward deviation. Incumbent 1's deviation will affect the auction outcome only when it prevents another bidder from dropping first at $P \in (B(v_1-\varepsilon), B(v_1))$. We only need to consider two possible cases: $v_1 \in (x, 1-x]$ and $v_1 \in (1-x, 1]$, as we have proved that conditional on dropping out, incumbents with values less than x will follow $\psi(\cdot)$.

2.1. $v_1 \in (x, 1-x]$. We will first consider the deviation of dropping at $B(v_1-\varepsilon) = 0$, and then consider the downward deviation of dropping at a price above zero, *i.e.*, $B(v_1-\varepsilon) \in (0, B(v_1))$.

2.1.a. $v_1-\varepsilon \leqslant x$. Let $\pi(v_1, \hat{v}_1)$ be the deviation payoff for R_1 by mimicking type $\hat{v}_1$. We have $\pi(v_1, v_1) > \pi(x, x)$. If $v_2 \leqslant x$, there is no profitable downward deviation to dropping at zero, as type x does not have an incentive to deviate downward, and that type v_1 and type x receive exactly the same payoff when deviating downward. If $v_2 > x$, the deviation will affect the outcome only if it prevents R_2 or E from dropping first at $P \in [0, B(v_1))$. If R_2 is the one being prevented, we have $v_2 \in (x, v_1)$ and $v_E \in (v_2-x, 1]$. The identities of the remaining bidders will change from R_1 and E to R_2 and E. The auction outcome can be affected in two ways: the outcome changes from R_1 winning against E to E winning against R_2 or from R_1 winning against E to R_2 winning against E. If bidder E is prevented from dropping first, it implies that $v_E \in [0, v_1-x)$ and $v_2 \in (v_E+x, 1]$. The identities of the remaining bidders will change from R_1 and R_2 to E and R_2. The outcome can be affected in two ways: the outcome changes from R_1 winning against R_2 to R_2 winning against E or from R_2 winning against R_1 to R_2 winning against E. Given all these, the change in R_1's expected payoff is given by

$$\int_{x}^{v_1}\int_{v_2+x}^{v_1+x}(-x-v_1+v_E)\,dv_E\,dv_2 + \int_{x}^{v_1}\int_{v_2-x}^{v_2+x}(v_E-v_1)\,dv_E\,dv_2 + \int_{0}^{v_1-x}\int_{v_E+x}^{v_1}(v_2-v_1)\,dv_2\,dv_E$$
$$= -\frac{1}{2}\int_{x}^{v_1}(v_1-v_2)\cdot(v_1-v_2+4x)\,dv_2 - \frac{1}{2}\int_{0}^{v_1-x}(-v_1+x+v_E)^2\,dv_E,$$

which is negative since $v_2 \leqslant v_1$. Therefore R_1 will be worse off from the deviation.

2.1.b. $B(v_1-\varepsilon) \in (0, B(v_1))$. If R_2 is prevented from dropping first, it implies that $v_2 \in (v_1-\varepsilon, v_1)$ and $v_E \in (v_2-x, 1]$. The identities of the remaining bidders will change from R_1 and E to R_2 and E. The auction outcome can be altered in two ways: the outcome changes from R_1 winning against E to E winning against R_2 or from R_1 winning against E to R_2 winning against E. If E is prevented from dropping first, it implies that $v_E \in (v_1-\varepsilon-x, v_1-x)$ and $v_2 \in (v_E+x, 1]$. The identities of the remaining bidders will change from R_1 and R_2 to E and R_2. The auction outcome can be altered in two ways: the outcome changes from R_1 winning against R_2 to R_2 winning against E or from R_2 winning against R_1 to R_2 winning against E, and R_1's payoff is not affected. Therefore, the change in R_1's expected payoff is given by

$$\int_{v_1-\varepsilon}^{v_1}\int_{v_2+x}^{v_1+x}(-x-v_1+v_E)\,dv_E\,dv_2 + \int_{v_1-\varepsilon}^{v_1}\int_{v_2-x}^{v_2+x}(v_E-v_1)\,dv_E\,dv_2 + \int_{v_1-\varepsilon-x}^{v_1-x}\int_{v_E+x}^{v_1}(v_2-v_1)\,dv_2\,dv_E$$

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$$= -\frac{1}{2}\int_{v_1-\varepsilon}^{v_1}(v_1-v_2)\cdot(v_1-v_2+4x)\,dv_2 - \frac{1}{2}\int_{v_1-\varepsilon-x}^{v_1-x}(-v_1+x+v_E)^2\,dv_E,$$

which is negative since $v_2 \leqslant v_1$. Therefore R_1 will be worse off from the deviation.

2.2. $v_1 \in (1-x, 1]$. We consider three cases in order.

2.2.a. $B(v_1-\varepsilon)=0$. If $v_2 \leqslant x$, based on the same arguments in 2.1.a. we can show that R_1 does not have an incentive to deviate. If $v_2 > x$, the deviation will affect the auction outcome only if it prevents R_2 or E from dropping first at $P \in [0, B(v_1))$. If R_2 is prevented from dropping first, it implies that either $v_2 \in (x, 1-x]$ and $v_E \in (v_2-x, 1]$, or $v_2 \in (1-x, v_1)$ and $v_E \in (2v_2-1, 1]$. The identities of the remaining bidders will change from R_1 and E to R_2 and E. The auction outcome can be affected in two ways: the outcome changes from R_1 winning against E to E winning against R_2 or from R_1 winning against E to R_2 winning against E. If E is prevented from dropping first, it implies that either $v_E \in [0, 1-2x)$ and $v_2 \in (v_E+x, 1]$, or $v_E \in (1-2x, 2v_1-1]$ and $v_2 \in (\frac{v_E+1}{2}, 1]$. The identities of the remaining bidders will change from R_1 and R_2 to E and R_2. The auction outcome can be altered in two ways: the outcome changes from R_1 winning against R_2 to R_2 winning against E or from R_2 winning against R_1 to R_2 winning against E. Therefore, the change in R_1's expected payoff from dropping at 0 instead of $B(v_1)$ is given by

$$\int_{x}^{1-x}\int_{v_2+x}^{1}(-x-v_1+v_E)\,dv_E\,dv_2 + \int_{x}^{1-x}\int_{v_2-x}^{v_2+x}(v_E-v_1)\,dv_E\,dv_2 + \int_{1-x}^{v_1}\int_{2v_2-1}^{1}(v_E-v_1)\,dv_E\,dv_2$$
$$+\int_{0}^{1-2x}\int_{v_E+x}^{v_1}(v_2-v_1)\,dv_2\,dv_E + \int_{1-2x}^{2v_1-1}\int_{\frac{v_E+1}{2}}^{v_1}(v_2-v_1)\,dv_2\,dv_E$$
$$= -\frac{1}{2}\int_{x}^{1-x}(v_1-v_2)\cdot(v_1-v_2+4x)\,dv_2 - 2\int_{1-x}^{v_1}(v_1-v_2)\cdot(1-v_2)\,dv_2$$
$$-\frac{1}{2}\int_{0}^{1-2x}(-v_1+x+v_E)^2\,dv_E - \frac{1}{8}\int_{1-2x}^{2v_1-1}(1-2v_1+v_E)^2\,dv_E,$$

which is clearly negative. Therefore, R_1 will be worse off from the deviation.

2.2.b. $B(v_1-\varepsilon) \in (0, 1-2x]$. Based on similar arguments as in case 2.1, the change in R_1's expected payoff is given by

$$\int_{v_1-\varepsilon}^{1-x}\int_{v_2+x}^{1}(-x-v_1+v_E)\,dv_E\,dv_2 + \int_{v_1-\varepsilon}^{1-x}\int_{v_2-x}^{v_2+x}(v_E-v_1)\,dv_E\,dv_2 + \int_{1-x}^{v_1}\int_{2v_2-1}^{1}(v_E-v_1)\,dv_E\,dv_2$$
$$+\int_{v_1-\varepsilon-x}^{1-2x}\int_{v_E+x}^{v_1}(v_2-v_1)\,dv_2\,dv_E + \int_{1-2x}^{2v_1-1}\int_{\frac{v_E+1}{2}}^{v_1}(v_2-v_1)\,dv_2\,dv_E$$
$$= -\frac{1}{2}\int_{v_1-\varepsilon}^{1-x}(v_1-v_2)\cdot(v_1-v_2+4x)\,dv_2 - 2\int_{1-x}^{v_1}(v_1-v_2)\cdot(1-v_2)\,dv_2$$
$$-\frac{1}{2}\int_{v_1-\varepsilon-x}^{1-2x}(-v_1+x+v_E)^2\,dv_E - \frac{1}{8}\int_{1-2x}^{2v_1-1}(1-2v_1+v_E)^2\,dv_E,$$

which is obviously negative. Therefore R_1 will be worse off from the deviation.

2.2.c. $B(v_1-\varepsilon) \in (1-2x, B(v_1))$. That $B(v_1-\varepsilon) > 1-2x$ implies that both incumbents can win against E. If R_2 is prevented from dropping first, it implies that $v_2 \in (v_1-\varepsilon, v_1)$ and $v_E \in (2v_2-1, 1]$. The identities of the remaining bidders will change from R_1 and E to R_2 and E. The deviation will change the auction outcome from R_1 winning against E to R_2 winning against E, and R_1's payoff changes from $v_1 - v_E$ to 0. If E is prevented from dropping first, it implies that $v_E \in (2(v_1-\varepsilon)-1, 2v_1-1]$ and $v_2 \in (\frac{v_E+1}{2}, 1]$. The identities of the remaining bidders will change from R_1 and R_2 to E and R_2. The auction outcome can be altered in two events: the outcome changes from R_1 winning against R_2 to R_2 winning against E, and R_1's payoff changes from $v_1 - v_2$ to 0; or from R_2 winning against R_1 to R_2 winning against E,

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and R_1's payoff is not affected. Putting together, if R_1 drops at $B(v_1-\varepsilon) > 1-2x$ instead of staying till $B(v_1)$, the change in his expected payoff equals

$$\int_{v_1-\varepsilon}^{v_1}\int_{2v_2-1}^{1}(v_E-v_1)\,dv_E\,dv_2+\int_{2(v_1-\varepsilon)-1}^{2v_1-1}\int_{\frac{v_E+1}{2}}^{v_1}(v_2-v_1)\,dv_2\,dv_E$$

$$=-2\int_{v_1-\varepsilon}^{v_1}(v_1-v_2)\cdot(1-v_2)\,dv_2-\frac{1}{8}\int_{2(v_1-\varepsilon)-1}^{2v_1-1}(1-2v_1+v_E)^2\,dv_E,$$

which is obviously negative. Therefore, when $v_1 \in (1-x, 1]$, R_1 can only be worse off from any downward deviation.

Next, we consider the case $x \geqslant 1/2$. The analysis will be similar to the case of $x < 1/2$ above.

1. Upward deviation. When the price has reached $B(v_1)$ and no bidder has dropped yet, we consider R_1's upward deviation to dropping at $B(v_1+\varepsilon)$. R_1's deviation will affect the auction outcome only if it allows another bidder to drop first at price $P \in (B(v_1), B(v_1+\varepsilon))$. We will consider three possible cases: $v_1 \in [0, 1-x)$, $v_1 \in [1-x, \frac{1}{2}]$, and $v_1 \in (\frac{1}{2}, 1)$.

1.1. $v_1 \in [0, 1-x)$. Suppose R_1 drops at $B(v_1+\varepsilon) > 0$ instead. We will consider two possibilities of v_2: $v_2 \in [0, \frac{1}{2}]$ and $v_2 \in (\frac{1}{2}, 1]$.

1.1.a. $v_2 \in [0, \frac{1}{2}]$. R_2 drops at zero. If R_1 follows $B(v_1) = 0$, the two Rs will tie at zero and the tie-breaking rule will let the bidder with higher value stay. If $v_2 < v_1$, both the tie-breaking rule and the deviation to $B(v_1+\varepsilon) > 0$ will make bidder 1 stay with E, and the auction will have the same outcome. If $v_2 \geqslant v_1$, the tie-breaking rule will make R_2 stay and the upward deviation will make R_1 stay. The auction outcome can be affected in two ways: the outcome changes from R_2 winning against E to E winning against R_1, and R_1's payoff changes from 0 to $-x$; or from R_2 winning against E to R_1 winning against E, and R_1's payoff changes from 0 to $v_1 - v_E$. Given these, R_1's change in expected payoff from dropping at $B(v_1+\varepsilon) > 0$ instead of $B(v_1) = 0$ is given by

$$\int_{v_1}^{1-x}\int_{v_1+x}^{v_2+x}(-x)\,dv_E\,dv_2+\int_{1-x}^{\frac{1}{2}}\int_{v_1+x}^{1}(-x)\,dv_E\,dv_2+\int_{v_1}^{\frac{1}{2}}\int_{0}^{v_1+x}(v_1-v_E)\,dv_E\,dv_2$$

$$=x\int_{v_1}^{1-x}(v_1-v_2)\,dv_2+x\int_{1-x}^{\frac{1}{2}}\bigl[v_1-(1-x)\bigr]\,dv_2+\frac{1}{2}\int_{v_1}^{\frac{1}{2}}\bigl(v_1^2-x^2\bigr)\,dv_2,$$

which is negative as all the integrands above are negative. Therefore R_1 will be worse off from the deviation.

1.1.b. $v_2 \in (\frac{1}{2}, 1]$. R_2 will drop at $2v_2 - 1 > 0$. When the current price is 0, R_1's deviation to $B(v_1+\varepsilon) > 0$, *i.e.*, $v_1+\varepsilon \in (\frac{1}{2}, v_1+x]$, instead of dropping at 0 will affect the auction outcome only when it allows another bidder to drop first at price $P \in [0, B(v_1+\varepsilon))$. If R_2 drops first after the deviation, it implies that $v_2 \in (\frac{1}{2}, v_1+\varepsilon)$ and $v_E \in (2v_2-1, 1]$. The two remaining bidders will change from R_2 and E to R_1 and E. The auction outcome could be affected in two ways: the outcome changes from R_2 winning against E to E winning against R_1, and R_1's payoff changes from 0 to $-x$; or from R_2 winning against E to R_1 winning against E, and R_1's payoff changes from 0 to $v_1 - v_E$. If instead bidder E drops first after the deviation, it implies that $v_E \in (0, 2(v_1+\varepsilon)-1)$ and $v_2 \in (\frac{v_E+1}{2}, 1]$. The two remaining bidders will change from R_2 and E to R_2 and R_1. In both cases R_2 will win and R_1's payoff will be zero. Given all these, R_1's change in expected payoff from dropping at $B(v_1+\varepsilon) > 0$ is given by

$$\int_{\frac{1}{2}}^{v_1+\varepsilon}\int_{v_1+x}^{1}(-x)\,dv_E\,dv_2+\int_{\frac{1}{2}}^{v_1+\varepsilon}\int_{2v_2-1}^{v_1+x}(v_1-v_E)\,dv_E\,dv_2$$

$$=\int_{\frac{1}{2}}^{v_1+\varepsilon}\bigl[v_1-(1-x)\bigr]\,dv_2+\frac{1}{2}\int_{\frac{1}{2}}^{v_1+\varepsilon}\bigl[v_1^2-x^2+(2v_1-2v_2+1)\cdot(1-2v_2)\bigr]\,dv_2,$$

which is negative (as can be easily verified). So R_1 will be worse off from the deviation.

1.2. $v_1 \in [1-x, \frac{1}{2}]$. Again we consider two possibilities of v_2: $v_2 \in [0, \frac{1}{2}]$, or $v_2 \in (\frac{1}{2}, 1]$.

1.2.a. $v_2 \in [0, \frac{1}{2}]$. If R_1 stays in the auction, he is certain to win against E. If $v_2 < v_1$, for the same reason as in the case $x < 1/2$, R_1's upward deviation to $B(v_1+\varepsilon) > 0$ will not affect the auction outcome. If $v_2 \in (v_1, \frac{1}{2}]$, the deviation to

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$B(v_1+\varepsilon)>0$ could change the auction outcome from R_2 winning against E to R_1 winning against E, and R_1's payoff changes from 0 to v_1-v_E. R_1's change in expected payoff from dropping at $B(v_1+\varepsilon)>0$ instead of $B(v_1)=0$ equals

$$\int_{v_1}^{\frac{1}{2}}\int_{0}^{1}(v_1-v_E)\,dv_E\,dv_2=\int_{v_1}^{\frac{1}{2}}\left(v_1-\frac{1}{2}\right)dv_E\,dv_2,$$

which is negative. Therefore R_1 will be worse off from the deviation.

1.2.b. $v_2\in(\frac{1}{2},1]$. R_2 will drop at $2v_2-1>0$. When the current price is 0, R_1's deviation to $B(v_1+\varepsilon)>0$, *i.e.*, $v_1+\varepsilon\in(\frac{1}{2},v_1+x]$, instead of dropping at 0 will affect the auction outcome only when it allows another bidder to drop first at the price of $P\in[0,B(v_1+\varepsilon))$. If R_2 drops first after the deviation, it implies that $v_2\in(\frac{1}{2},v_1+\varepsilon)$ and $v_E\in(2v_2-1,1]$. The two remaining bidders will change from R_2 and E to R_1 and E. The auction outcome could be changed from R_2 winning against E to R_1 winning against E, and R_1's payoff changes from 0 to v_1-v_E. If instead bidder E drops first after the deviation, it implies that $v_E\in(0,2(v_1+\varepsilon)-1)$ and $v_2\in(\frac{v_E+1}{2},1]$. The two remaining bidders will change from R_2 and E to R_2 and R_1. In both cases R_2 will win and R_1's payoff will be zero. Given all these, R_1's change in expected payoff from dropping at $B(v_1+\varepsilon)>0$ is given by

$$\int_{\frac{1}{2}}^{v_1+\varepsilon}\int_{2v_2-1}^{1}(v_1-v_E)\,dv_E\,dv_2=-2\int_{\frac{1}{2}}^{v_1+\varepsilon}(v_2-v_1)\cdot(1-v_2)\,dv_2,$$

which is negative because $v_2\geqslant v_1$ and $v_2\leqslant 1$. Hence bidder 1 will be worse off from the deviation.

1.3. $v_1\in(\frac{1}{2},1)$. When the price has reached $B(v_1)=2v_1-1>0$ and no bidder has dropped, it implies that both R_1 and R_2 could win against E, and R_2 has a higher value than R_1. We consider R_1's upward deviation by dropping at $B(v_1+\varepsilon)\in(2v_1-1,1]$. If R_2 drops first after the deviation, it implies that $v_2\in(v_1,v_1+\varepsilon)$ and $v_E\in(2v_2-1,1]$. The identities of the two remaining bidders will change from R_2 and E to R_1 and E. Such a deviation will change the outcome from R_2 winning against E to R_1 winning against E, and R_1's payoff changes from 0 to v_1-v_E. If E drops first after the deviation, it implies that $v_E\in(2v_1-1,2(v_1+\varepsilon)-1)$ and $v_2\in(\frac{v_E+1}{2},1]$. The identities of the two remaining bidders will change from E and R_2 to R_1 and R_2. In both cases R_2 will win and R_1's payoff will be zero. Given all these, R_1's change in expected payoff from dropping at $B(v_1+\varepsilon)$ instead of $B(v_1)$ is given by

$$\int_{v_1}^{v_1+\varepsilon}\int_{2v_2-1}^{1}(v_1-v_E)\,dv_E\,dv_2=2\int_{v_1}^{v_1+\varepsilon}(1-v_2)(v_1-v_2)\,dv_2,$$

which is negative as $v_1\leqslant v_2$ and $v_2\leqslant 1$. Therefore when $v_1\in[1-x,1)$, R_1 will not be better off from an upward deviation.

Finally, when $v_1=1$, R_1 will not stay at a price above 1 as doing so can only increase the chance of winning with a negative profit.

2. Downward deviation. When the price has reached $B(v_1-\varepsilon)$ with $v_1-\varepsilon\in[0,v_1)$ and no bidder has dropped yet, we consider R_1's downward deviation of dropping at $B(v_1-\varepsilon)$. R_1's deviation will affect the auction outcome only when it prevents another bidder from dropping first at $P\in(B(v_1-\varepsilon),B(v_1))$. We will focus on the case $v_1\in(\frac{1}{2},1]$ as this is the range of v_1 where a downward deviation is relevant. By following $B(\cdot)$, R_1 is supposed to drop at $2v_1-1>0$. We will first consider the deviation of dropping at $B(v_1-\varepsilon)=0$, and then consider the downward deviation of dropping at a positive price, *i.e.*, $B(v_1-\varepsilon)\in(0,B(v_1))$.

2.1. $B(v_1-\varepsilon)=0$. If $v_2\leqslant\frac{1}{2}$, R_1 and R_2 will both drop at price zero. The tie-breaking rule will let R_1, *i.e.*, the bidder with a higher value, stay in the auction. Therefore the deviation makes no difference to the auction outcome (R_1 bidding against E starting from price zero). If $v_2>\frac{1}{2}$, the deviation will affect the auction outcome only if it prevents R_2 or E from dropping first at $P\in[0,B(v_1))$. If R_2 is prevented from dropping first, it implies that $v_2\in(\frac{1}{2},v_1)$ and $v_E\in(2v_2-1,1]$. The identities of the remaining bidders will change from R_1 and E to R_2 and E. The auction outcome will change from R_1 winning against E to R_2 winning against E, and R_1's payoff changes from v_1-v_E to 0. If E is prevented from dropping first, it implies that $v_E\in[0,2v_1-1)$ and $v_2\in(\frac{v_E+1}{2},1]$. The identities of the remaining bidders will change from R_1 and R_2 to E and R_2. The auction outcome can be affected in two ways: the outcome changes from R_1 winning against R_2 to R_2 winning against E, and R_1's payoff changes from v_1-v_2 to 0; or from R_2 winning against R_1 to R_2 winning against E, and R_1's payoff is not affected. Given all these, R_1's change in expected payoff from dropping at 0 instead of $B(v_1)$ is given by

$$\int_{\frac{1}{2}}^{v_1}\int_{2v_2-1}^{1}(v_E-v_1)\,dv_E\,dv_2+\int_{0}^{2v_1-1}\int_{\frac{v_E+1}{2}}^{v_1}(v_2-v_1)\,dv_2\,dv_E$$

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$$= 2\int_{\frac{1}{2}}^{v_1}(1-v_2)(v_2-v_1)\,dv_2 - \frac{1}{8}\int_{0}^{2v_1-1}(1-2v_1+v_E)^2\,dv_E,$$

which is negative since $v_2 \leqslant v_1$. Therefore bidder 1 will be worse off from the deviation.

2.2. $B(v_1-\varepsilon) \in (0, B(v_1))$. When the price has reached $B(v_1-\varepsilon) > 0$, *i.e.*, $v_1-\varepsilon \in (\frac{1}{2}, v_1)$, we consider a deviation of dropping at $B(v_1-\varepsilon)$. The deviation will affect the auction outcome only if it prevents R_2 or E from dropping first at $P \in (B(v_1-\varepsilon), B(v_1))$. If R_2 is prevented from dropping first, it implies that $v_2 \in (v_1-\varepsilon, v_1)$ and $v_E \in (2v_2-1, 1]$. The identities of the remaining bidders will change from R_1 and E to R_2 and E. The auction outcome will change from R_1 winning against E to R_2 winning against E, and R_1's payoff changes from v_1-v_E to 0. If E is prevented from dropping first, it implies that $v_E \in (2(v_1-\varepsilon)-1, 2v_1-1)$ and $v_2 \in (\frac{v_E+1}{2}, 1]$. The identities of the remaining bidders will change from R_1 and R_2 to E and R_2. The auction outcome can be affected in two ways: the outcome changes from R_1 winning against R_2 to R_2 winning against E, and R_1's payoff changes from v_1-v_2 to 0; or from R_2 winning against R_1 to R_2 winning against E, and R_1's payoff is not affected. Given all these, R_1's change in expected payoff from dropping at $B(v_1-\varepsilon) \in (0, B(v_1))$ is given by

$$\int_{v_1-\varepsilon}^{v_1}\int_{2v_2-1}^{1}(v_E-v_1)\,dv_E\,dv_2 + \int_{2(v_1-\varepsilon)-1}^{2v_1-1}\int_{\frac{v_E+1}{2}}^{v_1}(v_2-v_1)\,dv_2\,dv_E$$

$$= 2\int_{v_1-\varepsilon}^{v_1}(1-v_2)\cdot(v_2-v_1)\,dv_2 - \frac{1}{8}\int_{2(v_1-\varepsilon)-1}^{2v_1-1}(1-2v_1+v_E)^2\,dv_E,$$

which is negative since $v_2 \leqslant v_1$. Therefore bidder 1 will be worse off from the deviation.

We have thus shown that, when $v_1 \in (\frac{1}{2}, 1]$, R_1 will be worse off from any downward deviation.

In summary, given that bidder 2 follows $B(\cdot)$ and bidder E bids his value, it is not profitable for bidder 1 to deviate (either upward or downward) from following $B(\cdot)$; hence the specified equilibrium is verified. □

A.2. Proof of Proposition 2

The derivation preceding to the proposition shows that the equilibrium has to satisfy the differential equations (1). We now argue that the boundary condition is given by $\beta(0) = \gamma(0) = 0$. That $\gamma(0) = 0$ is obvious given $\beta(0) = 0$. Now suppose $\beta(0) = \underline{b} > 0$, then we must have $\gamma(\underline{b}) = \underline{b}$: if $\gamma(\underline{b}) < \underline{b}$, there is no chance for the entrant of type $\underline{b}$ to win, so $\gamma(\underline{b}) \geqslant \underline{b}$; if $\gamma(\underline{b}) > \underline{b}$, type $\underline{b}$ entrant may win only to lose money, which is inconsistent with any equilibrium. Thus when $\beta(0) = \underline{b} > 0$, we must have $\gamma(\underline{b}) = \underline{b}$. However, given that R_2 follows β and E follows γ in which $\beta(0) = \gamma(\underline{b}) = \underline{b} > 0$, we claim that R_1 with $v_1 = 0$ will have an incentive to deviate from bidding $\beta(0) = \underline{b}$: by bidding $\underline{b}$, R_1 wins only if he wins the tie-break over R_2 in the event of $v_E \leqslant \underline{b}$ and $v_2 = 0$, in which case he incurs a net loss (due to the overbidding). By deviating to bidding at zero, R_1 will lose for sure and avoid a loss in such an event. For all the other events, R_1 is indifferent between bidding $\underline{b}$ and 0. As such, R_1 is strictly better off to deviate from bidding $\beta(0) = \underline{b}$ when $v_1 = 0$. This shows that in any symmetric increasing equilibrium, we must have the boundary condition $\beta(0) = \gamma(0) = 0$.

Next we show that $\beta(1) = \gamma(1) = \bar{b}$ for some $\bar{b} < 1$ and $\beta(v) > \gamma(v)$ for all $v \in (0, 1)$.

1. $\beta(1) = \gamma(1) = \bar{b}$ for some $\bar{b} < 1$.
   Clearly, $\gamma(1) \leqslant \beta(1)$ must hold because $\gamma(1) > \beta(1)$ is strictly dominated for an entrant with $v_E = 1$. Now suppose $\gamma(1) < \beta(1)$ holds in equilibrium. Let $\pi(v_1, \hat{v}_1)$ denote R_1's expected payoff when he bids $\beta(\hat{v}_1)$ given that his type is v_1 and that R_2 follows $\beta(\cdot)$. Then $\pi(v_1, \hat{v}_1)$ must be maximized at $\hat{v}_1 = v_1$. However, at $\beta(v_1) = \gamma(1)$, we can show that $\lim_{\hat{v}_1 \to v_1^-} \frac{\partial \pi(v_1,\hat{v}_1)}{\partial \hat{v}_1} < \lim_{\hat{v}_1 \to v_1^+} \frac{\partial \pi(v_1,\hat{v}_1)}{\partial \hat{v}_1}$, contradicting that $\pi(v_1, \hat{v}_1)$ achieves the maximum at $\hat{v}_1 = v_1$. Finally $\gamma(1) < 1$; otherwise the entrant with $v = 1$ has an incentive to underbid.
2. Incumbents bid more aggressively than the entrant.
   Whenever $\beta(v^*) = \gamma(v^*) = b^*$ and $v^* \neq 0$, from the second equation in the system (1), we have

$$\beta'(v^*) = \frac{2(v^*-b^*)}{v^*} < \frac{2(v^*-b^*+x)}{v^*} = \gamma'(v^*). \tag{4}$$

Note that the above inequality also holds at $v^* = 1$. So $\beta(1) = \gamma(1)$ and $\beta'(1) < \gamma'(1)$. Suppose the set $\{v \in (0,1): \beta(v) = \gamma(v)\}$ is non-empty, and let $v^{**} = \max\{v \in (0,1): \beta(v) = \gamma(v)\}$. It must be the case that $\beta'(v^{**}) > \gamma'(v^{**})$, which contradicts (4). Therefore, it has to be the case that $\beta(v) > \gamma(v)$ for all $v \in (0, 1)$. □

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